FORMAT OF THE QUESTION PAPER

Additional Mathematics Paper 1 (3472/1) is of the Graded Objective form. The candidates are required to give their responses by stating their own answers. There are 25 questions and the candidates are required to answer all the questions within 2 hours.

OVERALL PERFORMANCE

The candidates' achievement varies among the centres. Generally, the achievement of the candidates is satisfactory. The candidates' performance this year is better compared to that in 2006. The number of candidates obtaining a pass in the paper has increased.

Some of the candidates' achievement has measured up to the standard. However there are others who still do not have the necessary basic skills for the subject. They are still weak in the following topics: Logarithms, Vector, Trigonometry, Probability Distribution, Permutation and Combination.

PERFORMANCE ACCORDING TO ACHIEVEMENT GROUP

The High Achiever Group

Candidates have a good understanding of mathematical concepts and good basic mathematical skills. Candidates are able to identify the mathematical concepts that are to be applied and are able to give complete, accurate and precise answers. Candidates also possess high manipulative skills.
Candidates' working is organized, clear and systematic with the correct usage of methods and formulae. Hence, marking is easy and well done.
A variety of solution strategies were displayed by the candidates to solve certain items such as items on Functions (Question 3), Quadratic Equations (Question 4), Quadratic Functions (Questions 6), Progressions (Question 10), Circular Measure (Question 18) and Differentiation (Question 20).

However, there are some candidates in this group who are careless while transferring the correct answer to the answer space, misread the value given in the question or omit the negative sign in the final answer.

The Average Achiever Group

Candidates are able to answer accurately the easy items. The candidates' answers for the moderate and difficult items are incomplete and inaccurate since the values used in the manipulation are either wrongly approximated or miscalculated.

Candidates are able to identify the mathematical concepts that are to be applied but are unable to use the precise methods, symbols or language. They are either unsure of the solutions that they have written, thus giving a few different methods or they used an alternative method that is complicated and lengthy.

Candidates were able to use the formulae correctly but were careless when doing the calculation, algebraic manipulation and approximation. Candidates' working is less organised and systematic.

The Low Achiever Group

Candidates do not have strong basic mathematical skills. Calculation done is either incorrect or difficult to understand. Candidates do not understand what is required from the question and unable to master well the mathematical concepts.

Answers given are inaccurate or are not related to the requirements of the question. Candidates try to answer part of the question and some do not make any attempt at all.

Candidates' working is disorganized and difficult to understand. Majority of them do not show any working.

PERFORMANCE OF THE CANDIDATES IN DETAIL

QUESTION 1

Many candidates are able to obtain the answer in (a), i.e. $m = 3$.
They can also determine the relation between x and $h(x)$.
For example:

Answer / *Jawapan*: (*a*) $m =$ 3

(*b*) $h(x) = x + 1$

From the example, the candidates are able to write the relation using the correct function notation i.e. $h(x) = x + 1$.

Some candidates especially from the Average Achiever Group failed to write the relation using a complete function notation.
Example 1:

Answer / *Jawapan*: (*a*) $m =$ 3

(*b*) $h = x + 1$

The candidates have written $h = x + 1$ instead of $h(x) = x + 1$.
Example 2:

Answer / *Jawapan*: (*a*) $m =$ 3

(*b*) $x + 1$

The candidates have omitted '$h(x) =$'

Some candidates misinterpreted the domain and range of the function.

Example:

Answer / *Jawapan*: (*a*) $m = 3$

(*b*) $h(x) = x - 1$, $x \neq 1$

From the example above, the candidates have interchanged the domain and the range of the function. They assumed the relation is from $h(x) \to x$ instead of $x \to h(x)$.

QUESTION 2

Many students are able to identify that 5 is the image of *x* from the given *f*(*x*) = 5. Some of them are able to apply the concept of modulus to solve for the values of x.

Example:

$f(x) = |x - 3|$

$f(x) = 5$

$|x - 3| = 5$

$\pm(x - 3) = 5$

$+(x - 3) = 5$ $\quad$ $-(x - 3) = 5$

$x = 8$ $\quad$ $-x = 2$

$x = -2$

Answer / *Jawapan*: $x = -2, 8$

In this example, the candidates have replaced the modulus sign $|x - 3| = 5$ with $\pm(x - 3) = 5$ to solve for the correct values of x.

The following are the common mistakes made by the candidates.

Example 1:

$f(x) = |x - 3|$

$\therefore f(5) = |5 - 3|$

$\therefore f(5) = 2$

The value '5' is used as the object.

Example 2:

$f(x) = |x-3|$

$\cancel{x}\ f(x) = 5$

$|x-3| = 5$

$x = 5+3$

$x = 8$

Answer / *Jawapan*: $x =$ ......8......

Candidates only consider the positive value when the modulus sign is removed.

Example 3:

$f(x) = |x-3|$ $\qquad$ $\cancel{x = 5-3}$

$5 = |x-3|$ $\qquad$ $\cancel{x = 2}$

$x = |-3-5|$

$x = |-8|$

$\therefore\ x = |8|$

Answer / *Jawapan*: $x =$ ....

Candidates gave inaccurate answers.

Example 4:

$f(x) = |x-3|$

$5 = |x-3|$

$8 = |x|$

$x = \pm 8$

Answer / *Jawapan*: $x =$ ......± 8......

Candidates gave inaccurate answers.

Example 5:

$|x-3| = 5$

$x-3 = 25$

$x = 28$

Candidates tried to remove the modulus sign by squaring the equation on both sides. However in the process of doing so, they have forgotten to square the left- hand side of the equation.

QUESTION 3

Many students are able to determine the composite function of $h^2(x)$ accurately.

They understand that the value of *a* must be positive and that the value is used to find the value of *b*.

Example:

$a(ax+b)+b$

$h^2(x) = a^2x + ab + b.$

$h^2x = 36x - 35$

$a^2 = 36$

$a = \pm 6$

Since $a > 0$

$\therefore a = 6$

$b(a+1) = -35$

$7b = -35$

$b = -5$

In the example, the candidates have obtained the correct composite function $h^2(x) = a^2x + ab + b$ and compared it with the given $h^2(x) = 36x - 35$ to obtain $a^2 = 36$. Therefore $a = 6$ since $a \rangle 0$. Subsequently $a = 6$ is substituted into $b(a+1) = -35$, thereby obtaining the answer $b = -5$.

The following examples are the common mistakes made by the candidates.

Example 1:

Find the value of *a* and of *b*.
Cari nilai a dan nilai b.

$h: x \to ax+b$

$h(x) = ax+b$

$h^2(x) = hh(x)$

$= h[h(x)]$

$= h[ax+b]$

$= a(ax+b)+b$

$= a^2x+ab+b$

By comparing $h^2(x) = a^2x + ab + b$ and $h^2(x) = 36x - 35$

[3 marks]
[3 markah]

$a^2 = 36$

$a = \pm\sqrt{36}$

$a = 6$ or -6

When $a=6$,

$ab + b = -35$

$6(b)+b = -35$

$7b = -35$

$b = -\frac{35}{7}$

$= -5$

When $a = -6$

$ab+b = -35$

$(-6)(b)+b = -35$

$-5b = -35$

$b = 7$

Answer / *Jawapan*: $a =$ 6 and -6

$b =$ -5 and 7

3

Candidates gave two values of *a* even though question states that $a > 0$.

Example 2:

$h(x) = ax + b$

$hh(x) = 36x - 35$

$a(ax + b) + b = 36x - 35$

$a^2x + ba + b = 36x - 35$

Answer / *Jawapan*: $a =$ ....

$b =$ ....

Many candidates found the composite function $h^2(x)$ and equated it with $36x - 35$ and stopped at this stage.

Example 3:

$h(x) = ax + b$

$hh(x) = a(ax + b) + b = 36x - 35$

$a^2x + ab + b - 36x + 35 = 0$

$a^2x - 36x + ab + b + 35 = 0$

Candidates tried to solve for x instead of making comparison but failed to do so because there were too many unknowns in one equation.

Example 4:

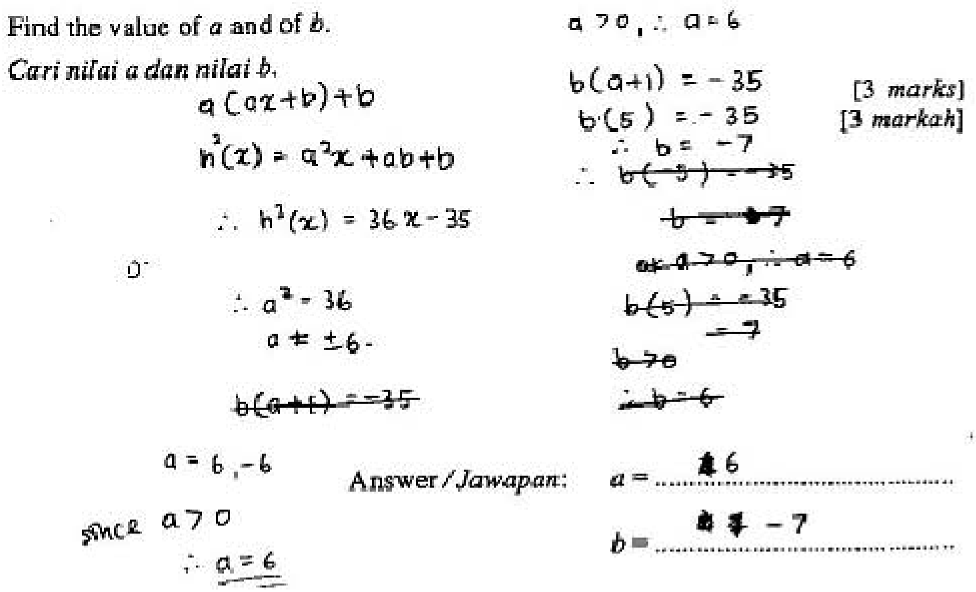
$a^2 = 36$ $\qquad ab + b = 35$

$a = 6$ $\qquad 7b = 35$

$b = 5$

Comparison is done carelessly. Candidates used $ab + b = 35$ instead of $ab + b = -35$

Example 5:



Find the value of a and of b.

Cari nilai a dan nilai b.

[3 *marks*]
[3 *markah*]

$a(ax+b)+b$

$h^2(x) = a^2x + ab + b$

$\therefore h^2(x) = 36x - 35$

$\therefore a^2 = 36$

$a = \pm 6$

~~$b(a+1) = -35$~~

$a = 6, -6$

since $a > 0$

$\therefore a = 6$

$a > 0, \therefore a = 6$

$b(a+1) = -35$

$b(5) = -35$

$\therefore b = -7$

~~$\therefore b(-5) = -35$~~

~~$b = 7$~~

~~$a > 0, \therefore a = 6$~~

~~$b(5) = -35$~~

~~$= -7$~~

~~$b > 0$~~

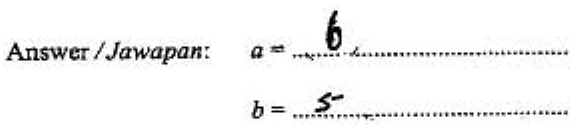
~~$\therefore b = 6$~~

Answer / *Jawapan*: $a = 6$

$b = -7$

Calculation is done carelessly. After obtaining $a = 6$, candidates made a mistake in $a+1=5$ instead of $a+1=7$. Hence, obtaining $b=-7$ instead of $b=-5$.

Example 6:

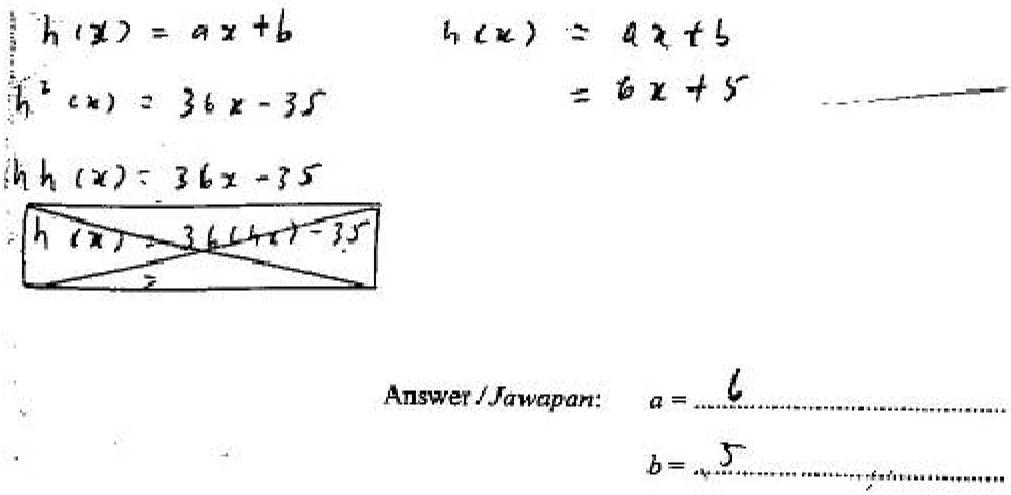


Answer / *Jawapan*: $a = 6$

$b = 5$

No working was shown when finding the values of *a* and *b*.

Example 7:



$h(x) = ax + b$

$h^2(x) = 36x - 35$

$hh(x) = 36x - 35$

$\boxed{\cancel{h(x) = 36(4x) - 35}}$

$h(x) = ax + b$

$= 6x + 5$

Answer / *Jawapan*: $a = 6$

$b = 5$

No conclusion at the end of working. Candidates did not give any conclusion at the end of working space. They must write at least $a = 6$ in the working space.

Example 8:

$h(x) = ax + b$

$h^2(x) = 36x - 35$

$h(x) = \sqrt{36x - 35}$

$= 6x - \sqrt{35}$

$a = 6 \quad b = \sqrt{35}$

Candidates do not understand the notation of composite functions. Candidates misinterpreted the composite function $h^2(x)$ as $(h(x))^2$.

Example 9:

$h^2(x) = (ax + b)(ax + b)$

$= a^2x + 2abx + b^2$

Candidates do not understand the notation of composite functions. They misinterpreted the composite function $h^2(x)$ as $(h(x))^2$.

QUESTION 4(a)

Many candidates are able to solve the quadratic equation by factorisation or by applying the formula.

Example 1:

a) $3x^2 + 5x - 2 = 0$

$(3x - 1)(x + 2) = 0$

$x = \frac{1}{3}, -2$

Here, the candidates have factorised the equation correctly.

Example 2:

$$3x^2+5x-2=0$$

$$x = \frac{-5 \pm \sqrt{5^2-4(3)(-2)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{25+24}}{6}$$

$$= \frac{-5 \pm \sqrt{49}}{6}$$

$$x = \frac{1}{3}, \quad -2$$

This example shows that the candidates are able to use the formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ correctly.

The following examples are the common mistakes made by the candidates.

Example1:

(a) $3x^2+5x-2=0$

$a=3 \quad b=5 \quad c=-2$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{5^2-4(3)(-2)}}{2(3)}$$

$$= \frac{-5 \pm 7}{6}$$

$$= \frac{-5+7}{6} \quad \text{or} \quad \frac{-5-7}{6}$$

$$= 0.33 \qquad = -2$$

Answers in decimal form are not rounded off to 4 significant figures.

Example 2:

$$3x^2+5x-2$$

$$(x+1)(3x-2)$$

$$x=-1, \quad x=\frac{2}{3}$$

Factorisation is done carelessly.

Example 3:

a) $3x^2 + 5x - 2 = 0$

$a = 3 \quad b = 5 \quad c = -2$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{1}}{6}$$

$$x = \frac{-5+1}{6} \qquad x = \frac{-5-1}{6}$$

$$= \frac{-4}{6} \qquad x = -1$$

$$x = -0.6667$$

4

4

Calculation is done carelessly. The $\sqrt{(5)^2 - -4(3)(-2)}$ should be $\sqrt{25+24} = \sqrt{49}$ instead of $\sqrt{25-24} = \sqrt{1}$.

Example 4:

a) $3x^2 + 5x - 2 = 0$

$(3x-1)(x+2) = 0$

$3x - 1 = 0$ or $x + 2 = 0$

$3x = 1 \qquad \underline{\underline{x = -2}}$

$\underline{\underline{x = \frac{1}{3}}}$

$3x^2 + 5x - 2 = 0$

$3x \quad 6x \quad -1$

$x \quad -x \quad 2$

$+5x$

$\therefore (3x-1)(x+2)$

Answer / *Jawapan*: (*a*) $x = \frac{1}{3}$ or $x = 2$

(*b*) $h = \frac{\sqrt{k}}{12}$

Answers were transferred carelessly to the answer space. The candidates should write $x = -2$ instead of $x = 2$ in the answer space.

Example 5:

$$3x^2 + 5x - 2 = 0$$
$$\left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 2 = 0$$

The candidates were careless in completing the square. They did not convert $3x^2 + 5x - 2 = 0$ to $x^2 + \frac{5}{3}x - \frac{2}{3} = 0$ before completing the square.

QUESTION 4(b)

Many candidates are able to translate the phrase 'two equal roots' as the condition $b^2 - 4ac = 0$. They can also apply the condition $b^2 - 4ac = 0$ to express *h* in terms of *k*.

Some candidates use the sum of roots (SOR) and the product of roots (POR) to find the relation between *h* and *k*.

Example 1:

b) $hx^2 + kx + 3 = 0$

$a = h \quad b = k \quad c = 3$

$b^2 - 4ac = 0$

$(k)^2 - 4(h)(3) = 0$

$k^2 - 12h = 0$

$k^2 = 12h$

$h = \frac{k^2}{12}$

Answer / *Jawapan*: (*a*) $x = -0.6667,\ x = -1$

(*b*) $h = \cancel{\frac{k^2}{12}}\ \frac{k^2}{12}$

From the example, the candidates substituted the correct values of a, b and c into $b^2 - 4ac = 0$ to obtain $h = \frac{k^2}{12}$.

Example 2:

$$hx^2 + kx + 3 = 0$$
$$x^2 + \frac{k}{h}x + \frac{3}{h} = 0$$
$$SOR = -\frac{k}{h} \qquad POR = \frac{3}{h}$$
$$\alpha + \alpha = -\frac{k}{h} \qquad \alpha^2 = \frac{3}{h}$$
$$\alpha = -\frac{k}{2h}$$
$$\therefore \left(-\frac{k}{2h}\right)^2 = \frac{3}{h}$$
$$\frac{k^2}{4h^2} = \frac{3}{h}$$
$$\frac{k^2}{12} = h$$

In this example, the candidates have used α to denote the root of the equation. They further used SOR $2\alpha = -\frac{k}{h}$ and POR , $\alpha^2 = \frac{3}{h}$ to obtain $h = \frac{k^2}{12}$.

The following examples are the common mistakes made by the candidates.

Example1:

a) $3x^2 + 5x - 2 = 0$
$(x - 0.33)(x + 2) = 0$
$x = 0.33 \quad x = -2$

b) $a = h \quad b = k \quad c = 3$
$b^2 - 4ac > 0$
$(k)^2 - 4(h)(3) > 0$
$k^2 - 12h > 0$
$-12h > -k^2$
$h > \frac{k^2}{12}$

Answer / ***Jawapan*****:** (*a*) $x = 0.33 \quad x = 2$

(*b*) $h = > \frac{k^2}{12}$

In this example, the candidates have used $b^2 - 4ac > 0$ to express h in terms of k.

Example 2:

b) $hx^2 + kx + 3 = 0$ [4 *marks*]
[4 *markah*]

$a = h \quad b = k \quad c = 3$

$\therefore$ ~~$b^2 - 4ac = 0$~~
~~$(k)^2 - 4(h)(3) = 0$~~
~~$k^2 - 12h = 0$~~

$b^2 - 4ac = 0$
$(k)^2 - 4(h)(3) = 0$
$k^2 - 12h = 0$
$-12h = \sqrt{k}$
$h = \frac{\sqrt{k}}{12}$

Calculation is done carelessly. Candidates wrote $-12h = \sqrt{k}$ instead of $-12h = -k^2$.

Example 3:

(b) $hx^2 + kx + 3 = 0$

$b^2 - 4ac = 0$

$k^2 - 4(h)(3) = 0$

$k^2 = 12h$

$k = \sqrt{12h}$

Answer / *Jawapan*: (*a*) 0.33, -2

(*b*) $h =$ ...., ...

Candidates misread the question's requirement. They have used k instead of h as the subject of the formula.

Example 4:

b) $x^2 + \frac{k}{h}x + \frac{3}{k} = 0$

$x^2 - \left(-\frac{k}{h}\right)x + \frac{3}{k} = 0$

let a = roots

$S.O.R = 2a = \frac{-k}{h}$

$P.O.R = a^2 = \frac{3}{k}$

$(S.O.R)\; a = \frac{-k}{2h}$, $a = \sqrt{\frac{3}{k}}$ (P.O.R)

$\frac{-k}{2h} = \sqrt{\frac{3}{k}}$

$2h = \frac{-k}{\left(\sqrt{\frac{3}{k}}\right)}$

$2h = \frac{-k}{\sqrt{3}}$

$h = \frac{-k}{2\sqrt{3}}$

Answer / *Jawapan*: (*a*) $x = \frac{1}{3}, -2$

(*b*) $h = \frac{-k}{2\sqrt{3}}$

The candidates misinterpreted the SOR and POR of the equation. In the example, the POR should be $\frac{3}{h}$ instead of $\frac{3}{k}$.

QUESTION 5

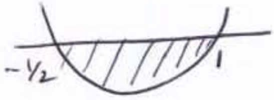
Many candidates are able to factorise accurately. Some can further use the 'graph sketching' or 'number line' method to find the range of values of x and able to select and shade the region that satisfies the inequality. They were also able to write the range using the correct inequality sign.

Example:

$2x^2 \le 1 + x$

$2x^2 - x - 1 \le 0$

$(2x+1)(x-1) \le 0$



$x = -\frac{1}{2}$, $x = 1$.

$\therefore -\frac{1}{2} \le x \le 1$

In this example, the candidates have used the graph sketching method correctly. They have indicated the region accurately and stated the range using the correct inequality sign i.e. $-\frac{1}{2} \le x \le 1$.

The following examples are the common mistakes made by the candidates.

Example 1:

$2x^2 \le 1 + x$

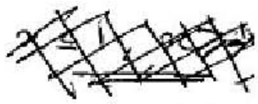
$2x^2 - x - 1 \le 0$

$(x-1)(2x+1) \le 0$

~~(i) $x < 0$~~

(i) $x \le 1$

(ii) $x \geq -1/2$ ✗



$x \le 1$, $x \geq$

Answer / *Jawapan*: ....

The candidates have not used the graph sketching or the number line method to obtain the correct range of values of x.

Example 2:

$2u^2 \le 1 + u$

$2u^2 - 1 - u \le 0$

$(2u-1)(u+1) \le 0$

$2u - 1 = 0$ $\quad$ $u + 1 = 0$

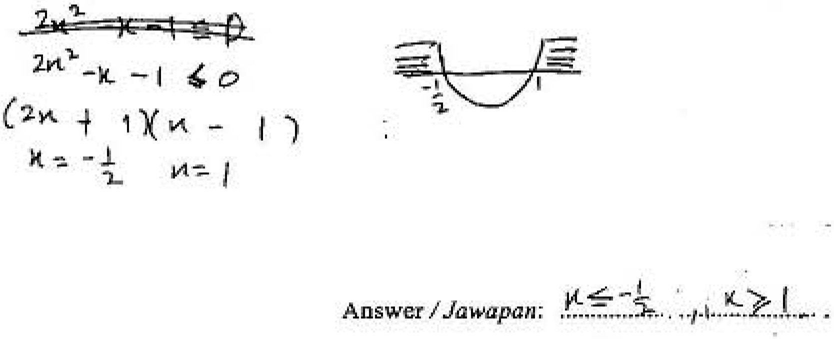
$2u = 1$ $\quad$ $u = -1$

$u = \frac{1}{2}$

Answer / *Jawapan*: ....

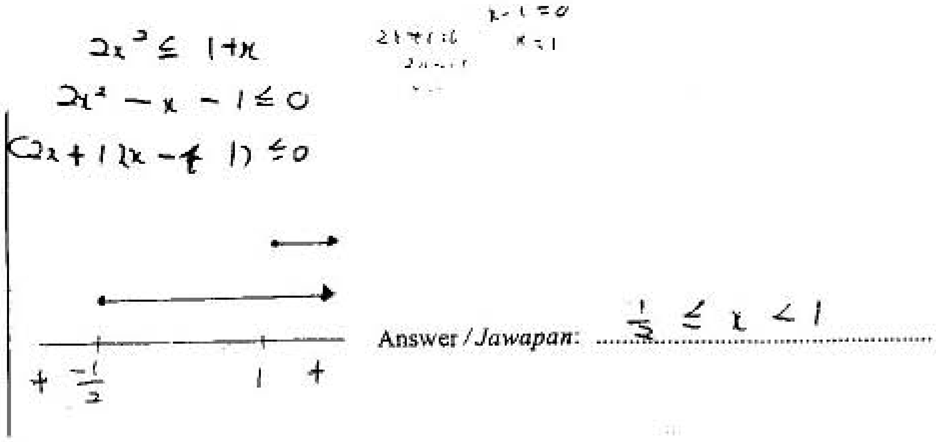
The candidates factorised $2x^2 - x - 1 \le 0$ as $(2x-1)(x+1) \le 0$ instead of $(2x+1)(x-1) \le 0$.

Example 3:



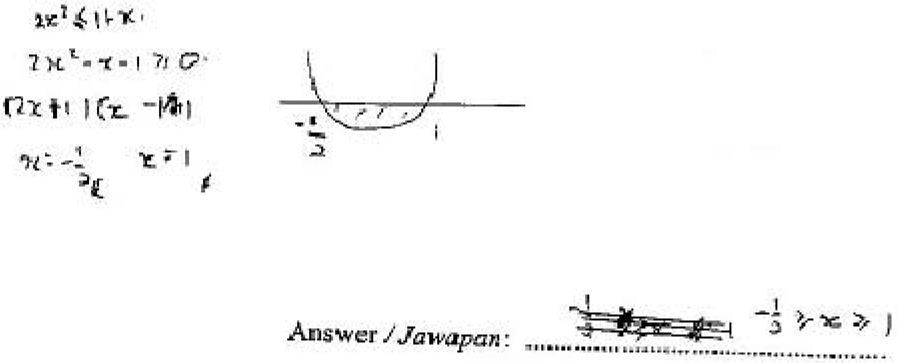
The candidates have shaded the wrong region.

Example 4:



The candidates have not indicated (by shading) the region that represent the range of values of x.

Example 5:



In this example, the candidates have indicated the region correctly but wrote the answer incorrectly. They should write $-\frac{1}{2} \le x \le 1$ instead of $-\frac{1}{2} \ge x \ge 1$.

Example 6:

$2x^2 \leq 1 + x$

$2x^2 - x - 1 \leq 0$

$(2x + 1)(x - 1) = 0$

$x = \frac{-1}{2}$, $x = 1$.

Answer / *Jawapan*: $x = -\frac{1}{2}$, $x = 1$

The candidates could only obtain the roots of the equation correctly. They were unable to continue to find the range of values of x.

QUESTION 6

The candidates in High Achiever Group can express $f(x) = x^2 + 2x - 4$ in the form of $f(x) = (x+m)^2 - n$ by completing the square correctly. They can further make the correct comparison to obtain the correct values of m and n.

Example 1:

$f(x) = (x^2 + 2x + (\frac{2}{2})^2 - (\frac{2}{2})^2 - 4)$

$= (x+1)^2 - 1 - 4$

$= (x+1)^2 - 5$

$\therefore m = 1$

$n = 5$

Answer / *Jawapan*: $m =$ 1

$n =$ 5

In the example, the candidates have expressed $f(x) = x^2 + 2x - 4$ as $f(x) = (x+1)^2 - 5$. They were able to make the correct comparison with $f(x) = (x+m)^2 - n$ and obtained the correct answer.

Yet, there are some candidates who are able to expand $f(x) = (x+m)^2 - n$ correctly. They can further make the correct comparison with $f(x) = x^2 + 2x - 4$ to obtain the correct values of m and n.

Example 2:

$$x^2 + 2x - 4 = (x+m)^2 - n$$
$$= x^2 + 2mx + m^2 - n$$

$\therefore \quad 2 = 2m \qquad , \qquad -4 = m^2 - n$

$\quad\ \ 1 = m \qquad\qquad\quad -4 = 1 - n$

$\qquad\qquad\qquad\qquad\quad -5 = -n$

$\qquad\qquad\qquad\qquad\quad\ \ 5 = n.$

In the example, the candidates have expanded $(x+m)^2 - n = x^2 + 2mx + m^2 - n$. They could further make the correct comparison with $f(x) = x^2 + 2x - 4$ which is $2 = 2m$ and $-4 = m^2 - n$. Hence obtained the correct answer $m = 1$ and $n = 5$.

The following examples are the common mistakes made by the candidates.

Example 1:

$f(x) = x^2 + 2x - 4$

$= [x^2 + 2x - (\frac{2}{2})^2 + (\frac{2}{2})^2 + 4]$

$= [(x-1)^2 + 1 + 4]$

$f(x) = (x-1)^2 + 5$

compare :

$f(x) = (x+m)^2 - n$

$\therefore m = -1 \quad , \quad n = 5$

$x + m = 0 \qquad x - 1 = 0$

$x = -m \qquad x = 1$

$-m = 1$

$m = -1$

$(x-1)^2 - 3$

Answer / *Jawapan*: $m = -1$

$n = 5$

Mistakes made in expressing $f(x) = x^2 + 2x - 4$ in the form $f(x) = (x+m)^2 - n$. The candidates wrote $f(x) = (x-1)^2 + 5$ instead of $f(x) = (x-1)^2 - 5$.

Example 2:

$$f(x) = x^2 + 2x - 4$$
$$= \left[x^2 + 2x + \left(\tfrac{2}{2}\right)^2 - \left(\tfrac{2}{2}\right)^2\right] - 4$$
$$= \left[(x+1)^2 - 1\right] - 4$$
$$= (x+1)^2 - 1 - 4$$
$$\cancel{= (x+1)^2}$$
$$= (x + 1)^2 - 5$$
$$m \qquad n$$

Answer / *Jawapan*: $m = 1$

$n = -5$

Comparison was done carelessly. The candidates compared n with -5 instead of $-n$ with -5.

Example 3:

$$x^2 + 2x - 4 = (x+m)^2 - n$$
$$= x^2 + 2mx + m^2 - n$$
$$2mx + m^2 = 2 \quad , \quad -n = -4$$

Some candidates were unable to make the correct comparison. They should compare $2m$ with 2 and $m^2 - n$ with -4.

Example 4:

$$x^2 + 2x - 4 = (x+m)^2 - n$$
$$x^2 + 2x - 4 = x^2 + 2xm + 2m - n$$

Answer / *Jawapan*: $m = 1$

$n =$

Calculation was done carelessly. The candidates made mistake while expanding $f(x) = (x+m)^2 - n$. They should have written $x^2 + 2mx + m^2 - n$.

Example 5:

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2(1)}$$

$$= -3.236 \quad , \quad 1.236$$

Some candidates tried to solve the equation $x^2 + 2x - 4 = 0$ by using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

QUESTION 7

The candidates in the High Achiever Group can express $\log_4 \left(\dfrac{8b}{c}\right)$ in terms of x and y.

Example:

$$\log_4 \left(\frac{8b}{c}\right)$$

$$= \log_4 8b - \log_4 c$$

$$= \frac{\log_2 8b}{\log_2 4} - \frac{\log_2 c}{\log_2 4}$$

$$= \frac{\log_2 8b}{\log_2 2^2} - \frac{\log_2 c}{\log_2 2^2}$$

$$= \frac{\log_2 8b - \log_2 c}{2}$$

$$= \frac{\log_2 8 + \log_2 b - \log_2 c}{2}$$

$$= \frac{\log_2 2^3 + \log_2 b - \log_2 c}{2}$$

$$= \frac{3 + x - y}{2}$$

$$= \frac{x - y + 3}{2}$$

Answer / *Jawapan*: $\dfrac{x - y + 3}{2}$

In the above example, the candidates could apply the product and quotient laws of logarithm and change the base correctly.

However, there are some candidates who are unable to apply the product law correctly.

Example:

$$\log_4\left(\frac{8b}{c}\right) = \log_4 8b - \log_4 c$$

$$= \frac{\log_2 8b}{\log_2 4} - \frac{\log_2 c}{\log_2 4}$$

$$= \frac{8x - y}{\log_2 4}$$

$$= \frac{8x - y}{\log_2 2^2}$$

$$= \frac{8x - y}{2}$$

Answer / *Jawapan*: $\frac{8x-y}{2}$

In this example, the candidates failed to see that '8b' is a product of '8' and '*b*'. Hence, they were unable to obtain the correct answer.

Most of the candidates cannot combine both laws of logarithm and change the base to obtain the correct answer.

Example 1:

$$\log_4 \frac{8b}{c} = \frac{\log_4 8b}{\log_4 c}$$

$$= \frac{\log_4 8 + \log_4 b}{\log_4 c}$$

$$= \frac{\log_{\cancel{4}2} 2^3 + \log_2 b}{\log_2 c}$$

$$= \frac{3 + x}{y}$$

In this example, the candidates could only apply the product law correctly.

Example 2:

$$\log_4 \frac{8b}{c} = \log_4 8b - \log_4 c$$

$$= \log_2 \left(\frac{8b}{4}\right) - \log_2 \left(\frac{c}{4}\right)$$

$$= \frac{8x - y}{4}$$

In this example, the candidates could only apply the quotient law correctly.

Example 3:

$$\log_2 b = x$$

$$\log_2 c = y$$

$$\log_4 \left(\frac{8b}{c}\right) = \frac{\log_2 \frac{8b}{c}}{\log_2 4}$$

$$= \frac{\log_2 \frac{8b}{c}}{\log_2 2^2}$$

In this example, the candidates could only change the base correctly.
From the examples given, most candidates can either apply the product law or the quotient law or change base correctly.

Some candidates are careless in writing the final answer.

Example:

$$\log_4 \left(\frac{8b}{c}\right)$$

$$= \log_4 8b - \log_4 c$$

$$= \log_4 8 + \log_4 b - \log_4 c$$

$$= \frac{\log_2 8}{\log_2 2^2} + \frac{\log_2 b}{\log_2 2^2} - \frac{\log_2 c}{\log_2 2^2}$$

$$= \frac{\log_2 8 + \log_2 b - \log_2 c}{2}$$

$$= \frac{(3 + x - y)}{2} \times 2$$

$$= 3 + x - y$$

Here the candidates multiplied the answer by 2 and obtained an incorrect final answer.

QUESTION 8

Many candidates are able to change the base of the indices correctly. Subsequently, they are able to apply the laws of indices accurately and managed to obtain the answer by equating the indices.

Example:

$$9\left(\frac{3^n}{3}\right) = 3^{3n}$$

$$3(3^n) = 3^{3n}$$

$$3^{n+1} = 3^{3n}$$

$$n+1 = 3n$$

$$1 = 2n$$

$$n = \frac{1}{2}$$

Answer / *Jawapan*: $n = \frac{1}{2}$

From the example above, the candidates were able to apply the following laws of indices: $\left(a^m\right)^n = a^{mn}$ and $a^{m-n} = \dfrac{a^m}{a^n}$.

However, there are some candidates who are careless in their calculation.

Example:

$$9(3^{n-1}) = 27^n$$

$$(3^3)(3^{n-1}) = \cancel{(3^3)(3^3)}\,(3^3)^n$$

$$\cancel{3^{3+n-1}}$$

$$3^{3+n-1} = (3^3)^n$$

$$3+n-1 = 3n$$

$$2+n = 3n$$

$$2 = 3n - n$$

$$2 = 2n$$

$$\frac{2}{2} = n$$

$$1 = n \quad \#$$

Answer / *Jawapan*: $n = 1$

In this example, the candidates wrote 9 as 3^3 instead of 3^2.

Some candidates apply the wrong concept while changing the number to the same base.

Example:

$$9(3^{n-1}) = 27^n$$
$$27^{n-1} = 27^n$$
$$n-1 = n$$

$$9(3^{n-1}) = 27^n$$
$$27^{9n-9} = 27^n$$
$$9n-9 = n$$
$$\cancel{9n = n+9}$$
$$8n = 9$$
$$n = \frac{9}{8}$$

Answer / *Jawapan*: $n =$ 9/8

From the example above, the candidate multiplied 9 by 3^{n-1} and obtained the wrong base 27^{n-1} instead of 3^{2+n-1}.

Some candidates apply incorrectly the product law of indices.

Example:

$$\cancel{9(3^{n-1}) = 27^n}$$
$$\cancel{3^2(3^{n-1}) = (3^3)^n}$$
$$\cancel{3^{2n-2} = 3^{3n}}$$
$$\cancel{\therefore 2n-3 = 3n}$$
$$\cancel{2n-3n = 3}$$
$$\cancel{-n = 3}$$

$$9(3^{n-1}) = 27^n$$
$$3^2(3^{n-1}) = (3^3)^n$$
$$3^{2n-2} = 3^{3n}$$
$$\therefore 2n-2 = 3n$$
$$2n-3n = 2$$
$$-n = 2$$
$$n = -2$$

Answer / *Jawapan*: $n =$ −2

In the above example, the candidates multiplied the indices instead of adding them i.e. $3^2\left(3^{n-1}\right) = 3^{2n-2}$ instead of $3^2\left(3^{n-1}\right) = 3^{2+n-1}$.

There are candidates who tried to solve the equation by taking logarithm on both sides.

Example:

$$\log 9(3^{n-1}) = \log 27^n$$
$$\log 9 \cdot \log 3^{n-1} = n \log 27$$
$$\log 9 \cdot (n-1) \log 3 = n \log 27$$
$$(n-1) \log 27 = n \log 27$$
$$n-1 = n$$

Answer / *Jawapan*: $n =$ ....

From the example above, the candidate failed to apply the product laws of logarithm correctly. The candidate should have written $\log 9 + (n-1)\log 3 = n\log 27$ instead of $\log 9.(n-1)\log 3 = n\log 27$

QUESTION 9

Many candidates are able to give the correct answer in (a) i.e. geometric progression. They are able to define or describe the characteristic of the geometric progression.
Example:

(a) $d = 8x - 16x$
$= -8x$
$d = 4x - 8x$
$= -4x$
$\therefore$ the common difference are not the same.

$r = \dfrac{4x}{8x}$
$= \dfrac{1}{2}$
$r = \dfrac{8x}{16x}$
$= \dfrac{1}{2}$

$\therefore$ the ratio is the same / constant.
$\therefore$ Geometric progression

(b) The sequence are geometric progression because the common ratio is constant that is $\frac{1}{2}$

Answer / *Jawapan*: (*a*) Geometric Progression

(*b*) the common ratio is constant, r is $\frac{1}{2}$

From the example, the candidates were able to show the working involving calculation of ratio between two consecutive terms and state the common ratio, $r = \dfrac{1}{2}$.

However, there are some candidates who failed to give a valid and comprehensive reason.

Example:

a) $\dfrac{8x}{16x} = \frac{1}{2}x$ ② $\dfrac{4x}{8x} = \frac{1}{2}x$

$\therefore$ The sequence is a geometric progression.

b) The reason is because the value when the sequence is divided is the same. $T_1, T_2, T_3 \ldots$

Example : $\dfrac{T_2}{T_1} = r$, $\dfrac{T_3}{T_2} =$

Answer / *Jawapan*: (*a*) Geometric progression.

(*b*) ....

From the example, the candidates did not give the value of the common ratio which is $\frac{1}{2}$.

A few candidates are confused with the terms 'common difference' and 'common ratio'.

Example:

$r = \frac{8x}{16x} = 0.5$

Answer / *Jawapan*: (*a*) Geometric progression common difference

(*b*) h is constant = 0.5

In the example, the candidates should write the common ratio = $\frac{1}{2}$ instead of common difference = $\frac{1}{2}$.

Some candidates made mistakes or were careless when they simplify the fraction.
Example:

$\frac{8x}{16x} = \frac{1}{2}x$, $\frac{4x}{8x} = \frac{1}{2}x$

$\because$ since $r = \frac{1}{2}x$

$\therefore$ geometric progression

Answer / *Jawapan*: (*a*) Geometric progression

(*b*) The ratio is $\frac{1}{2}x$

In this example, the candidates should write $\frac{8x}{16x} = \frac{1}{2}$, $\frac{4x}{8x} = \frac{1}{2}$,

QUESTION 10

Most of the candidates can apply the basic characteristic of an arithmetic progression $d = T_n - T_{n-1}$ to form an algebraic equation. Some can further form an equation by using the concept of common difference in an arithmetic progression in order to solve for x.

Example:

$$T_2 - T_1 = T_3 - T_2$$
$$8-(5-x) = 2x-8$$
$$8-5+x = 2x-8$$
$$8-5+8 = 2x-x$$
$$x = 11$$

Answer / *Jawapan*: ....

From the example, many candidates did not give the value of common difference, *d*, instead they stopped at $x = 11$.

Some candidates think that x and the common ratio are the same.

Example:

$$(8) - (5-x) = (2x) - (8)$$
$$3 + x = 2x - 8$$
$$2x - x = 3+8$$
$$x = 11 \quad \therefore d = 11$$

Answer / *Jawapan*: $d = 11$

From the example above, some of the candidates took the value $x = 11$ as the answer for the common difference.

Many candidates were careless in algebraic manipulation.

Example:

$$8 - 5 - x = 2x - 8$$
$$3 - x = 2x - 8$$
$$3 + 8 = 2x + x$$
$$11 = 3x$$
$$\frac{11}{3} = x$$
$$x = \frac{11}{3}$$

$$5 - \frac{11}{3} = \frac{4}{3}$$
$$d = 8 - \frac{4}{3}$$
$$= \frac{20}{3}$$

Answer / *Jawapan*: $\frac{20}{3}$

From the example above, most candidates did not bracket the term $5-x$. Hence obtaining the wrong algebraic expression '$3-x$' instead of '$3+x$'.

There are some candidates who have completely no understanding of the concept of arithmetic progression.

Example:

$$(5-x)8 = 8(2x)$$
$$40 - 8x = 16x$$
$$40 = 16x - 8x$$
$$40 = 8x$$
$$\frac{40}{8} = x$$
$$5 = x$$

$$8 - 2(5)$$
$$= 8 - 10$$
$$= -2$$

Answer / *Jawapan*: -2

In this example, the candidates have multiplied two consecutive terms to obtain the common difference.

Some candidates can apply the concept of geometric progression to solve for the value of x.

Example:

$$\frac{T_2}{T_1} = \frac{T_3}{T_4}$$
$$\frac{8}{5-x} = \frac{2x}{8}$$
$$64 = 10x - 2x^2$$
$$2x^2 - 10x + 64 = 0$$
$$x^2 - 5x + 32 = 0$$
$$(x-8)(x-4) = 0$$

From the example, the candidates have applied the concept of common ratio '$\frac{8}{5-x} = \frac{2x}{8}$' to find the value of x instead of the concept of common difference '$8-(5-x) = 2x-8$'.

QUESTION 11

Majority of candidates know the first term of geometric progression i.e. a = 27. They are also able to apply the concept of common ratio and the sum to infinity of geometric progression correctly.

Example:

GP = 27, 18, 12

$a = 27$

$r = \frac{18}{27}$

$= \frac{6}{9}$

$S_\infty = \frac{a}{1-r}$

$= \frac{27}{1-\left(\frac{6}{9}\right)}$

$= \frac{27}{\frac{3}{9}}$

$= 27 \times \frac{9}{3}$

$\underline{\underline{S_\infty = 81}}$

From the example, the candidates knew that the first term, $a = 27$ and the common ratio, $r = \frac{18}{27}$. They were able to substitute these values into the formula $S_\infty \approx \frac{a}{1-r}$ in order to obtain 81 as the correct answer.

However, there are some candidates who used the wrong formula to find the value of r. This result in the wrong answer obtained for S_∞ even though the candidates know its correct formula.

Example:

$T_3 = 27, 18, 12$

$S_\infty = \frac{a}{1-r}$, $|r| < 1$

$r = \frac{27}{18}$

$= \frac{3}{2}$

$S_\infty = \frac{27}{1-\frac{3}{2}}$, $|r| < 1$

$= \frac{27}{-0.5}$

$= -54$

Answer / *Jawapan*: $S_\infty = |-54|$

$S_\infty = 54$

From the example above, the candidates used $r = \frac{T_1}{T_2} = \frac{27}{18}$ instead of $r = \frac{T_2}{T_1} = \frac{18}{27}$.

Some candidates are careless in the mathematical operation involving fractions even though they have managed to substitute the correct values of a and r into the correct formula S_∞.

Example:

$\therefore r = \frac{18}{27}$ $\quad r = \frac{2}{3}, \; a = 27$

$= \frac{2}{3}$

$\therefore S_\infty = \frac{a}{1-r}, \; |r| < 1$

$= \frac{27}{1-\frac{2}{3}}$

$= \frac{27}{\frac{1}{3}}$

$= 9$

From the example above, the candidates have divided 27 by 3 instead of multiplying by 3 for $\frac{27}{\frac{1}{3}}$.

Premature approximation is used to obtain the value of r. Hence, obtaining the incorrect value of S_∞ when the value of r is substituted.

Example:

$r = \frac{18}{27} = 0.67$

$S_\infty = \frac{a}{1-r}$

$= \frac{27}{1-0.67}$

$= \frac{27}{0.33}$

$= 81.8$

The candidates should have rounded off the value of r to 4 significant figures i.e. 0.6667 instead of 0.67 .

Yet, there are some students who are unable to determine the correct value of r and also used the wrong formula for S_∞ .

Example:

$S_\infty = \frac{a}{r-1} \quad |r| > 1$

$\frac{27}{1.5-1}$

$= \frac{27}{0.5}$

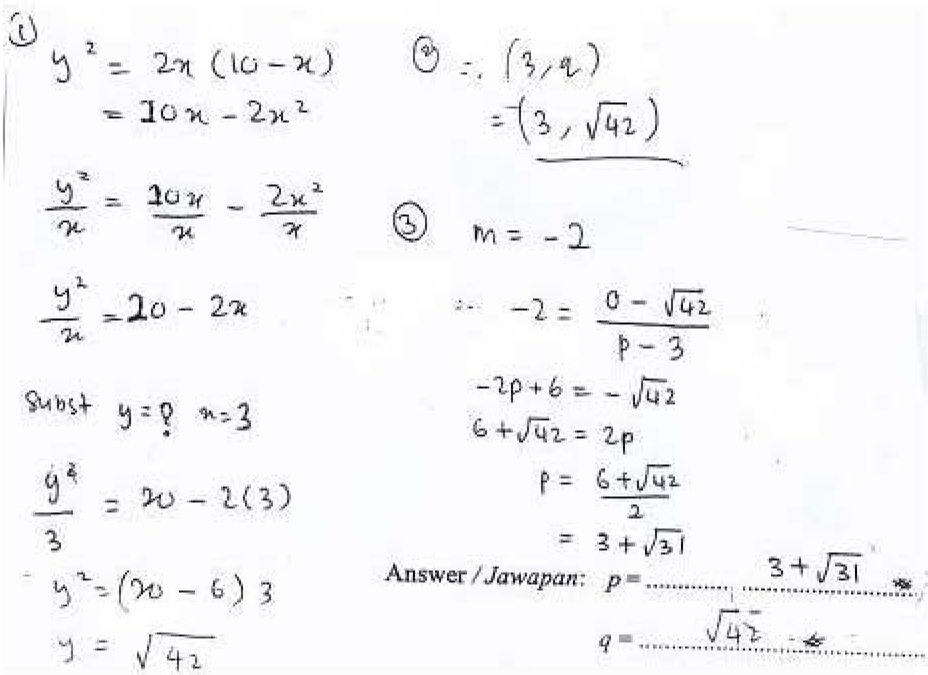
$\infty = 54$

From the example above, the candidates have substituted the incorrect value of '$r = 1.5$' into the incorrect formula $S_\infty \approx \dfrac{a}{r-1}$.

QUESTION 12

Majority of the candidates did not perform well in this question. They can convert the equation from the non-linear form to its linear form. They can also determine the value of the gradient from the equation in its linear form and use the formula of the gradient to find the value of p.

Example 1:


(1)

$y^2 = 2x(10-x)$

$= 10x - 2x^2$

$\dfrac{y^2}{x} = \dfrac{10x}{x} - \dfrac{2x^2}{x}$

$\dfrac{y^2}{x} = 20 - 2x$

Subst $y = ?$ $x = 3$

$\dfrac{y^2}{3} = 20 - 2(3)$

$y^2 = (20-6)3$

$y = \sqrt{42}$

(2) $\therefore (3, q)$

$= (3, \sqrt{42})$

(3) $m = -2$

$\therefore -2 = \dfrac{0-\sqrt{42}}{p-3}$

$-2p + 6 = -\sqrt{42}$

$6 + \sqrt{42} = 2p$

$p = \dfrac{6+\sqrt{42}}{2}$

$= 3 + \sqrt{31}$

Answer / *Jawapan*: $p =$ $3+\sqrt{31}$

$q =$ $\sqrt{42}$

From the example above, the candidates could obtain the equation in the linear form i.e. $\dfrac{y^2}{x} = 20 - 2x$. However, they failed to see that $\dfrac{y^2}{3}$ is actually q. Instead they substituted $x = 3$ into the formula to find the value of q. This results in the wrong value of p obtained even though the gradient formula

$$-2 = \frac{0-q}{p-3}$$

$$-2 = \frac{0-\sqrt{42}}{p-3}$$

is used correctly.

Example 2:

$$y^2 = 2x(10-x)$$

$$\frac{y^2}{x} = \frac{20x}{x} - \frac{2x^2}{x}$$

$$\frac{y^2}{x} = 20-2x$$

$$q = 20$$

$$\frac{20-0}{3-p} = -2$$

$$20-0 = -2(3-p)$$

$$20 = -6+2p$$

$$26 = 2p$$

$$p = 13$$

Answer / *Jawapan*: p = ...13......

q = ...20......

In this example, the candidates could obtain the equation in its linear form i.e. $\frac{y^2}{x} = 20 - 2x$. However, they mistook 8 as y – intercept of the equation which is 20. They also obtained the wrong answer value of p even though the gradient formula

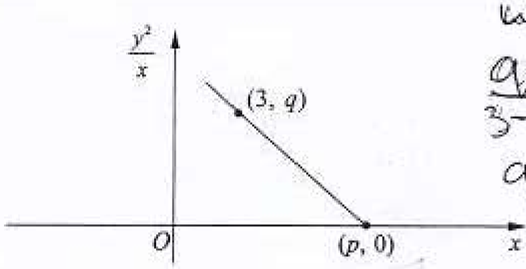
$$\frac{q-0}{3-p} = -2$$

$$\frac{20-0}{3-p} = -2$$

is used correctly.

Some candidates were careless when they expand the non-linear equation given.

Example:



when $p = 5$

$\frac{q-0}{3-5} = -4$

$q - 0 = 8$

$q = 8$ #

Diagram 2
Rajah 2

Find the value of p and of q.
Cari nilai p dan nilai q.

[3 *marks*]
[3 *markah*]

$y^2 = 2x(10-x)$

$y^2 = 20x - 4x^2 \div x$

$\frac{y^2}{x} = 20 - 4x$

$Y = mX + C$

$Y = \frac{y^2}{x}$ $m = -4$ $C = 20$

$\Rightarrow y = -4x + 20$

point p when $y = 0$

$y = -4x + 20$

$0 = -4x + 20$

$-20 = -4x$

$x = \frac{-20}{-4}$

$x = 5$ #

Answer / *Jawapan*: $p =$ 5

$q =$ 8

In this example, the candidates expanded $y^2 = 2x(10-x)$ and obtained $y^2 = 20x - 4x^2$ instead of $y^2 = 20x - 2x^2$. The candidates have substituted the correct values $x = p$ and $y = 0$ into the their wrong linear equation $y = -4x + 20$, hence obtained $p = 5$. Subsequently, the candidates used the incorrect value of the gradient -4 instead of -2 to obtain q

i.e $\quad \frac{q-0}{3-p} = -4$

$$\frac{q-0}{3-5} = -4$$

Yet, many candidates substituted the point $(3,q)$ and $(p,0)$ into the non-linear equation given.

Example:

$q^2 = 2(3)(10-3)$
$= 6(7)$
$= 42$
$= 7$

$0^2 = 2(p)(10-p)$
$0 = 2p(10-p)$
$0 = 20p - 4p^2$
$p = 5$

Answer / *Jawapan*: $p = $ ....5..........

$q = $ ....7..........

In this example, the candidates substituted $x = 3$, $y = q$ and $x = p$, $y = 0$ into the non-linear equation $y^2 = 2x(10 - x)$, hence obtained the answer $p = 5$ $q = 7$.

QUESTION 13

Majority of the candidates were able to state the value of *h* = 2 by using *h* = *y*-intercept. Some of them could further apply the relation of the gradients of two parallel lines to find the value of *k*.

Example:

$\cancel{\frac{y}{h} = 1 - \frac{x}{6}}$

y

$\frac{x}{6} + \frac{y_{\times 3}}{2_{\times 3}} = 1$

$-\frac{1}{3}$

$x + 3y = 6$

$3y = -x + 6$

$\boxed{y = -\frac{1}{3}x + 2}$

$y = -kx$

$\frac{x}{6} + \frac{y}{2} = 1$

$x + 3y = 6$

$y = -\frac{1}{3}x + 2$

$-k = -\frac{1}{3}$

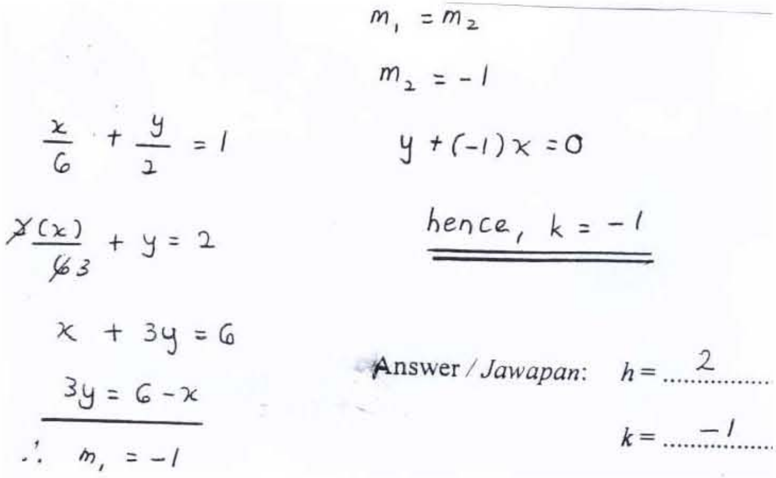
Answer / *Jawapan*: $h = $ ....2..........

$k = $ .... $1/3$ ....

From the example above, the candidates could recognise h as the $y-$intercept i.e. $h = 2$.

The candidates were able to convert the equation $\frac{x}{6} + \frac{y}{2} = 1$ from the intercept form to its gradient form $y = -\frac{1}{3}x + 2$. They can also convert the equation $y + kx = 0$ from the general form to its gradient form $y = -kx$. They were able to relate the gradients of two parallel lines by equating the corresponding gradient, $-k = -\frac{1}{3}$, and hence obtained the value $k = \frac{1}{3}$.

There are candidates who can recognise h as the $y-$ intercept, i.e. $h = 2$. They are also able to relate the gradients of two parallel lines correctly by equating the two corresponding gradients. However, they fail to convert the equation of the straight line from the intercept form to the gradient form $y = -\frac{1}{3}x + 2$ correctly. Example:



$\frac{x}{6} + \frac{y}{2} = 1$

$\frac{\cancel{2}(x)}{\cancel{6}3} + y = 2$

$x + 3y = 6$

$3y = 6 - x$

$\therefore m_1 = -1$

$m_1 = m_2$

$m_2 = -1$

$y + (-1)x = 0$

hence, $k = -1$

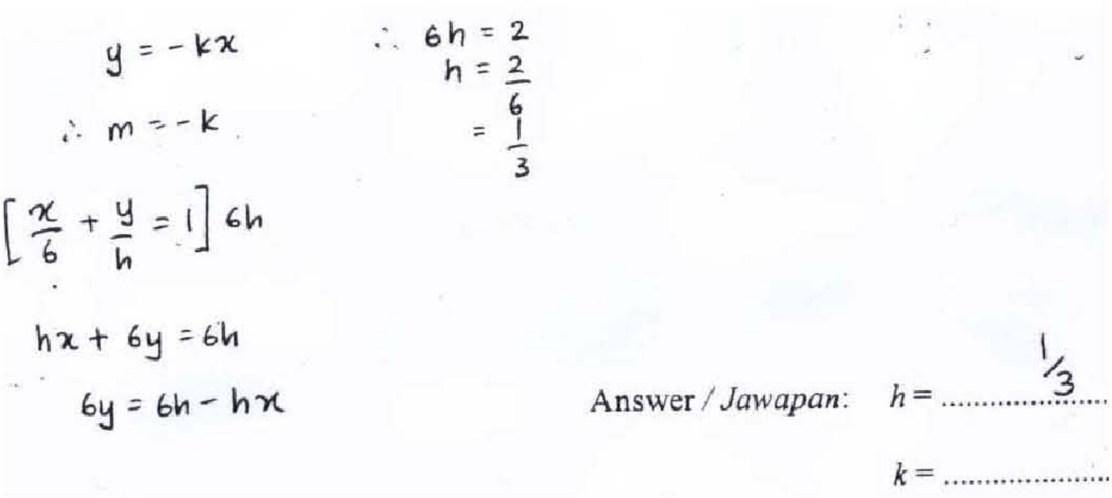
Answer / *Jawapan*: $h =$ 2

$k =$ -1

From this example, the candidates have converted $\frac{x}{6} + \frac{y}{2} = 1$ to $3y = 6 - x$ which should be $y = 2 - \frac{1}{3}x$.

Some candidates try to convert the equation given into the gradient form and equate incorrectly its $y-$ intercept with 2.

Example:



$y = -kx$

$\therefore m = -k$

$\left[\frac{x}{6} + \frac{y}{h} = 1\right] 6h$

$hx + 6y = 6h$

$6y = 6h - hx$

$\therefore 6h = 2$

$h = \frac{2}{6}$

$= \frac{1}{3}$

Answer / *Jawapan*: $h =$ 1/3

$k =$

In this example, the candidates should have written $y = h - \frac{h}{6}x$ instead of $6y = 6h - hx$ as the gradient form. Hence, by equating $6h = 2$, the candidates obtained the wrong answer for h.

QUESTION 14

Many candidates were able to use the formula for area of triangle, A = $\frac{1}{2}\begin{vmatrix} 5 & 4 & p & 5 \\ 2 & 6 & -2 & 2 \end{vmatrix}$ accurately and subsequently expand it to the form $\frac{1}{2}\left|(30 - 8 + 2p) - (8 + 6p - 10)\right| = 30$. However many candidates could only obtain one correct answer because they do not know the concept that if $|x| = a$, then x = a and x = – a .

Example:

$$\text{Area of triangle} = \frac{1}{2}\left|(x_1y_2 + x_2y_3 + x_3y_1) - (x_2y_1 + x_3y_2 + x_1y_3)\right|$$

$$30 = \frac{1}{2}\left|\left[(5\times6) + (4\times-2) + (p\times\cancel{2})\right] - \left[(4\times2) + (p\times6) + (5\times-2)\right]\right|$$

$$30 = \frac{1}{2}\left|(30 - 8 + 2p) - (8 + 6p - 10)\right|$$

$$30 = \frac{1}{2}\left|(22 + 2p) - (6p - 2)\right|$$

$$30 = \frac{1}{2}\left|(22 + 2p - 6p + 2)\right|$$

$$30 = \frac{1}{2}\left|(-4p + 24)\right|$$

$$30 = \frac{1}{\cancel{2}}(\cancel{-4}^{\,2}p + \cancel{24}^{\,12})$$

Answer / *Jawapan*: $p =$ −9

$$30 = -2p + 12$$

$$-2p = 30 - 12$$

$$-2p = 18$$

$$-p = \cancel{18}$$

In the example above, the candidates did not consider the negative value when they omit the modulus sign i.e. $30 = -(-2p + 12)$. Hence, they obtained one answer only which is $p = -9$ instead of $p = -9$, $p = 21$.

There are some candidates who have managed to expand A = $\frac{1}{2}\begin{vmatrix} 5 & 4 & p & 5 \\ 2 & 6 & -2 & 2 \end{vmatrix}$ to the form $\frac{1}{2}\left|(30 - 8 + 2p) - (8 + 6p - 10)\right| = 30$ but were careless in the mathematical operation in the subsequent steps to solve for p .

Example:

$$\frac{1}{2}\begin{vmatrix} 5 & 4 & p & 5 \\ 2 & 6 & -2 & 2 \end{vmatrix} = 30$$

$$\frac{1}{2}\,|30 + (-8) + 2p - (-10) - 6p - 8| = 30$$

$$\frac{1}{2}\,|30 - 8 + 2p + 10 - 6p - 8| = 30$$

$$\frac{1}{2}(24 - 4p) = 30$$

$$24 - 4p = 15$$

$$-4p = 15 - 24$$

$$-4p = -9$$

$$p = \frac{9}{4}$$

Answer / *Jawapan*: $p = \frac{9}{4}$

Here, the candidates have divided 30 by 2 instead of multiplying by 2.

There are also some candidates who have omitted $\frac{1}{2}$ from the correct formula.

Example:

$$\text{Area of triangle} = \begin{vmatrix} 5 & 4 & p & 5 \\ 2 & 6 & -2 & 2 \end{vmatrix}$$

$$30 = |30 - 8 + 2p - 8 + 6p + 10|$$

$$30 = |-4p + 24|$$

$$30 = -4p + 24$$

$$4p = -6$$

$$p = -\frac{3}{2}$$

Answer / *Jawapan*: $p = -3/2$

In this example, the candidates have done all the mathematical operations correctly but did not obtain the correct answer since '$\frac{1}{2}$' before the modulus have been omitted.

QUESTION 15

Many candidates can find the vector $\overrightarrow{OB}$ as an addition of vectors $\overrightarrow{OA}$ and $\overrightarrow{AB}$. However, there are others who give the vector $\overrightarrow{OB}$ as an addition of other vectors i.e. $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BD}$

Some are able to apply the ratio concept accurately to find the vector $\overrightarrow{OD}$

Example:

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= 9\underset{\sim}{x} + 5\underset{\sim}{y}$

$\overrightarrow{OD} = \frac{3}{4}\overrightarrow{OB}$

$= \frac{3}{4}[9\underset{\sim}{x} + 5\underset{\sim}{y}]$

$= \frac{27}{4}\underset{\sim}{x} + \frac{15}{4}\underset{\sim}{y}$.

Answer / *Jawapan*: $\overrightarrow{OD}$ = $\frac{27}{4}\underset{\sim}{x} + \frac{15}{4}\underset{\sim}{y}$

In this example, the candidates have obtained $\overrightarrow{OB}$ using $\overrightarrow{OA}$ + $\overrightarrow{AB}$ and have applied the correct ratio i.e. $\overrightarrow{OD} = \frac{3}{4}\overrightarrow{OB}$ to find $\overrightarrow{OD}$.

Most of the candidates could obtain $\overrightarrow{OB}$ correctly but unable to apply the correct ratio to find $\overrightarrow{OD}$.

Example:

(1) $\overrightarrow{OB} = \overrightarrow{OC} + \overrightarrow{CB}$

$= 5\underset{\sim}{y} + 9\underset{\sim}{x}$

(2) $\overrightarrow{DB} = \frac{1}{3}(\overrightarrow{OB})$

$= \frac{1}{3}(5\underset{\sim}{y} + 9\underset{\sim}{x})$

$= \frac{5}{3}\underset{\sim}{y} + 3\underset{\sim}{x}$

(3) $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CB} + \overrightarrow{BD}$

$= \overrightarrow{OC} + \overrightarrow{CB} + (-\overrightarrow{DB})$

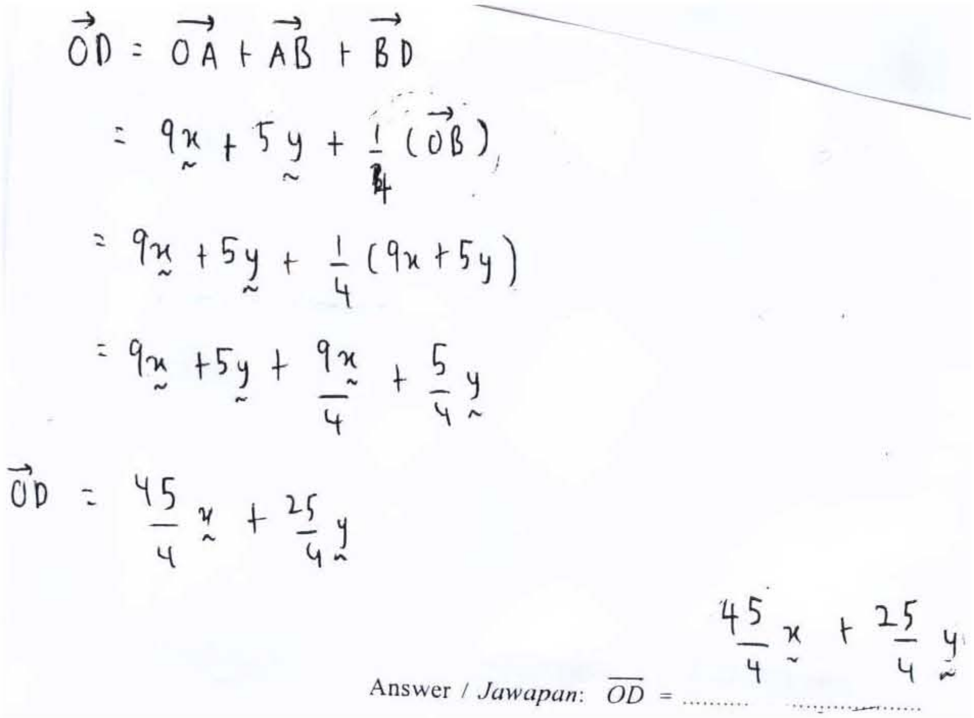
$= 5\underset{\sim}{y} + 9\underset{\sim}{x} + (-\frac{5}{3}\underset{\sim}{y} - 3\underset{\sim}{x})$

Answer / *Jawapan*: $\overrightarrow{OD}$ = $6\underset{\sim}{x} + \frac{10}{3}\underset{\sim}{y}$

The candidates have used the incorrect ratio to find $\overrightarrow{DB}$. They have used $\overrightarrow{DB} = \frac{1}{3}\overrightarrow{OB}$ instead of $\overrightarrow{DB} = \frac{1}{4}\overrightarrow{OB}$.

Some candidates also made mistakes in determining the direction of the vector.

Example:



$\vec{OD} = \vec{OA} + \vec{AB} + \vec{BD}$

$= 9\underset{\sim}{x} + 5\underset{\sim}{y} + \frac{1}{4}(\vec{OB})$

$= 9\underset{\sim}{x} + 5\underset{\sim}{y} + \frac{1}{4}(9x + 5y)$

$= 9\underset{\sim}{x} + 5\underset{\sim}{y} + \frac{9\underset{\sim}{x}}{4} + \frac{5}{4}\underset{\sim}{y}$

$\vec{OD} = \frac{45}{4}\underset{\sim}{x} + \frac{25}{4}\underset{\sim}{y}$

Answer / *Jawapan*: $\overline{OD} = \frac{45}{4}\underset{\sim}{x} + \frac{25}{4}\underset{\sim}{y}$

In this example, the candidates have represented $\overrightarrow{BD} = \frac{1}{4}\overrightarrow{OB}$ instead of $\overrightarrow{BD} = \frac{1}{4}\overrightarrow{BO}$

QUESTION 16

Majority of the candidates are able to find the vector $2\underline{a} - \underline{b}$ correctly. Some of them can use the formula $\hat{r} = \frac{\underline{r}}{|\underline{r}|}$ to find the unit vector in the direction of $2\underline{a} - \underline{b}$.

Example:

(a) $2\begin{pmatrix}2\\8\end{pmatrix}-\begin{pmatrix}-1\\4\end{pmatrix}$

$\begin{pmatrix}4\\16\end{pmatrix}-\begin{pmatrix}-1\\4\end{pmatrix}$

$\begin{pmatrix}5\\12\end{pmatrix}$

(b) $\dfrac{5\underset{\sim}{i}+12\underset{\sim}{j}}{\sqrt{5^2+12^2}}$

$=\dfrac{5\underset{\sim}{i}+12\underset{\sim}{j}}{\sqrt{169}}$

$=\dfrac{5\underset{\sim}{i}+12\underset{\sim}{j}}{13}$

Answer / *Jawapan*: (*a*) $\begin{pmatrix}5\\12\end{pmatrix}$ or $5\underset{\sim}{i}+12\underset{\sim}{j}$

(*b*) $\dfrac{5\underset{\sim}{i}+12\underset{\sim}{j}}{13}$

In this example, the candidates obtained the vector $2\underline{a}-\underline{b}$ by manipulating $2\begin{pmatrix}2\\8\end{pmatrix}-\begin{pmatrix}-1\\4\end{pmatrix}$ correctly. They have also used the formula of unit vector i.e. $\dfrac{5i+12j}{\sqrt{5^2+12^2}}$ to obtain the correct answer.

However, there are a few candidates who do not know that the final answer has to be in form of $\begin{pmatrix}x\\y\end{pmatrix}$ or $xi+yj$.

Example:

a) $2\underset{\sim}{a}-\underset{\sim}{b}=2\begin{pmatrix}2\\8\end{pmatrix}-\begin{pmatrix}-1\\4\end{pmatrix}$

$=\begin{pmatrix}4\\16\end{pmatrix}-\begin{pmatrix}-1\\4\end{pmatrix}$

$=\begin{pmatrix}5\\12\end{pmatrix}$

$=5\underset{\sim}{a}+12\underset{\sim}{b}$

b) unit vector $2\underset{\sim}{a}-\underset{\sim}{b}$

$=\dfrac{5\underset{\sim}{a}+12\underset{\sim}{b}}{\sqrt{25+144}}$

$=\dfrac{5a+12\underset{\sim}{b}}{\sqrt{169}}$

$=\dfrac{5a+12b}{13}$

$=\dfrac{1}{13}(5\underset{\sim}{a}+12\underset{\sim}{b})$

Answer / *Jawapan*: (*a*) $5\underset{\sim}{a}+12\underset{\sim}{b}$

(*b*) $\frac{1}{13}(5\underset{\sim}{a}+12\underset{\sim}{b})$.

Here, the candidates have managed to obtain $2\underline{a}-\underline{b}=\begin{pmatrix}5\\12\end{pmatrix}$ but gave the final answer as $5\underline{a}+12\underline{b}$. Subsequently, the candidates' answer in (b) is incorrect even though the candidates have used the correct formula to find the unit vector in the direction of $2\underline{a}-\underline{b}$.

QUESTION 17

Generally, only the candidates from the High Achiever Group can answer this question correctly.

Example:

$\cot x + 2\cos x = 0$

$\dfrac{\cos x}{\sin x} + 2\cos x = 0$

$\dfrac{\cos x + 2\cos x \sin x}{\sin x} = 0$

$\dfrac{\cos x(1 + 2\sin x)}{\sin x} = 0$

$\cos(1 + 2\sin x) = 0$

when $\cos x = 0$

$x = 90°,$

$270°$

$1 + 2\sin x = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

basic $x = -30°$

$x = 210°, 330°$

Answer / *Jawapan*: $90°, 270°, 210°, 330°$.

In this example, the candidates have used $\cot x = \dfrac{1}{\tan x}$ and formed a complete trigonometric equation . They have factorised the equation correctly. They have equated $\cot x = 0$, $1 + 2\sin x = 0$ to find the values of x .

The following examples are the common mistakes made by the candidates.

Example 1:

$\cot x + 2\cos x = 0$

$\dfrac{1}{\tan x} + 2\left(\dfrac{\sin x}{\tan x}\right) = 0$

$(\times \tan x)$ $\quad 1 + 2\sin x = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

base angle $= 30°$

3rd quadrant $= 180° + 30°$

$= 210°$ #

4th quadrant $= 360° - 30°$

$= 330°$ #

Answer / *Jawapan*: $210°, 330°$

Here, the candidates have multiplied the equation by $\tan x$, resulting in an equation with one factor only i.e. $1 + 2\sin x = 0$. Therefore, the candidates can only give the values of x that satisfy $1 + 2\sin x = 0$.

Example 2:

$\cot x + 2\cos x = 0$

$\dfrac{\cos x}{\sin x} + 2\cos x = 0$

$\cos x \left(\dfrac{1}{\sin x}\right) + 2\cos x = 0$

$\therefore \cos x \left(\dfrac{1}{\sin x} + 2\right) = 0$

$\therefore \cos x = 0$

$x = 90^\circ, 270^\circ$

$\therefore \dfrac{1}{\sin x} + 2 = 0$

$\dfrac{1}{\sin x} = -2$

$\sin x = -\dfrac{1}{2}$

$x = (360^\circ - 30^\circ), (270^\circ - 30^\circ)$

$= 330^\circ, 240^\circ$

$\therefore x = 90^\circ, 270^\circ, 330^\circ, 240^\circ$

Answer / *Jawapan*: $x = 90^\circ, 270^\circ, 330^\circ, 240^\circ$

In this example, the candidates have used $270^0 - 30^0$ instead of $180^0 + 30^0$ to find the angle in the third quadrant.

QUESTION 18

Most candidates are able to use the formulae $s = r\theta$ to find the arc length and $A = \frac{1}{2}r^2\theta$ to find the area of sector. They are able to use the Pythagoras Theorem to find the height of triangle OAD and subsequently use the formula $A = \frac{1}{2} \times base \times height$ to find its area. Some candidates use the formula for area of triangle OAD = $\frac{1}{2}(5)^2 \sin 1.85 rad$. Many candidates are aware that the Area of shaded region = Area of sector OBC – Area of triangle of OAD

Example:

~~$\dfrac{1.85 \times 180}{\pi} = 106^\circ$~~

~~arc length $s = r\theta$~~

~~$s = 10(106)$~~

~~$= 1060$ cm~~

~~area of AOD $= \frac{1}{2} \times 5 \times 8$~~

~~$= 20$ cm^2~~

~~area of BOC $= \frac{1}{2}(100)(106)$~~

~~$= 5300$ cm^2~~

~~area of shaded region $= 5300 - 20$~~

~~$= 5280$ cm^2~~

a) arc length BC, $s = r\theta$

$= 10(1.85)$

$= 18.5$ cm

b) area of AOD $= \frac{1}{2} \times 3 \times 8$

$= 12$ cm^2

area of BOC $= \frac{1}{2} \times 100 \times 1.85$

$= 92.5$ cm^2

area of shaded region $= 92.5 - 12$

$= \cancel{70.8}\ 80.5$ cm^2

Answer / *Jawapan*: (*a*) ~~1060 cm~~ 18.5 cm

(*b*) ~~5280 cm^2~~ 80.5 cm^2

In this example, the candidates have used the formula $s = r\theta$ correctly for part (a). They have obtained the height of the triangle OAD by applying Pythagoras Theorem, and subsequently substituted the values: base = 8, height = 3 into the formula $A = \frac{1}{2} \times \text{base} \times \text{height}$. Hence, the candidates have obtained the area of the shaded region by using Area of sector OBC – Area of triangle of OAD i.e. 92.5 – 12.

Many candidates have mistook the shaded region to be the area of segment.

Example:

Arc BC = $s = r\theta$

$s = 10(1.85\text{rad})$

$= 18.5\text{ cm}$

$1.85 \times \frac{180}{\pi} = 106^\circ$

Area of shaded region

$= \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2}(10)^2(1.85 - \sin 106^\circ)$

$= \cancel{50(0.8887)}$

$= \cancel{44.44\text{ cm}^2}$

$= 50(0.89)$

$= 44.50$

Answer / *Jawapan*: (*a*) 18.5cm

(*b*) $\cancel{44.44\text{cm}^2}$ 44.50 cm^2

In the example, the candidates have used the formula $\frac{1}{2}r^2(\theta - \sin\theta)$ to find the area of shaded region.

Many candidates are unable to find the area of triangle OAD correctly.

Example 1:

~~a) length of arc = radius × angle~~

$\cancel{s = r\theta}$

$\cancel{= 5 \times 1.85 \text{ rad}}$

a) $\cancel{= 9.25 \text{ rad}}$

b) Area of sector $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 5^2 \times \cancel{1.85}\ 106$

$= \cancel{23.125}$

$= 2000$ cm

Area of triangle $= \frac{1}{2} bh$

$= \frac{1}{2} \times 8 \times 5$

$= 20$ cm

a) $s = r\theta$

1.85 rad ⇒ degree

1 rad $= \frac{180}{\pi}$

$= \frac{180}{\pi} \times 1.85$

$= 106°$

$s = r\theta$

$= 5 \times 106°$

$= 530$ cm

∴ shaded region

$= 2000 - 20$

$= 1980 \text{ cm}^2$

Answer / *Jawapan*: (*a*) 530 cm.

(*b*) 1980 cm^2

In this example, the candidates have substituted the value of θ in degrees instead of in radians to find the area of sector OAB. They also substituted the value $h = 5$ instead of $h = 3$ into the formula $A = \frac{1}{2} \times \text{base} \times \text{height}$.

Example 2:

a) $S = r\theta$

$S = 10(1.85)$

$= 18.5$ cm.

b) Area: $\frac{1}{2} r^2 \theta$

$\frac{1}{2}(10)^2(1.85) - \frac{1}{2}(5)^2(1.85)$

$= 92.5 - 23.13$

$= 69.38 \text{ cm}^2$

Answer / *Jawapan*: (*a*) 18.5 cm.

(*b*) 69.38 cm^2

Here the candidates should have used $\frac{1}{2}(5)(5)\sin 1.85$ instead of $\frac{1}{2}(5)(5)(1.85)$ to find the area of triangle OAD.

QUESTION 19

Most candidates know that $\frac{dy}{dx} = 3kx + 5$ is the gradient function. Some are able to equate $\frac{dy}{dx}$ to 9 when $x = 2$.

Example:

$$\frac{dy}{dx} = 3kx + 5$$

$$9 = 3(2)k + 5$$

$$9 = 6k + 5$$

$$\frac{2}{3} = k$$

Answer / *Jawapan*: $k = \frac{2}{3}$

In the example above, the candidates have substituted $x = 2$ into the correct equation $3kx + 5 = 9$ in order to solve for the value of k which is $\frac{2}{3}$.

The following examples are the common mistakes made by the candidates.

Example 1:

$$y = f(x)$$

$$y = \frac{dy}{dx}$$

$$y = 3kx + 5$$

$$= 3(2)k + 5$$

$$= 6k + 5$$

$$-5 = 6k$$

$$-\frac{5}{6} = k$$

Answer / *Jawapan*: $k = -\frac{5}{6}$

In this example, the candidates have substituted $x = 2$ into the equation $\frac{dy}{dx} = 3kx + 5$ but failed to know that $\frac{dy}{dx} = 9$. Instead they equated $3k(2) + 5$ to 0, thereby obtaining the wrong answer, $k = -\frac{5}{6}$.

Example 2:

$$\frac{dy}{dx} = 3kx + 5$$
$$y = \frac{3kx^2}{2} + 5x$$
$$9 = \frac{3k(2)^2}{2} + 5(2)$$
$$9 = 6k + 10$$
$$-1 = 6k$$
$$-\frac{1}{6} = k$$

In the example above, the candidates have integrated $\frac{dy}{dx} = 3kx + 5$ to obtain the equation $y = 3k\frac{x^2}{2} + 5x$. Subsequently, they substituted $x = 2$ and $y = 9$ into the equation and obtained the wrong answer $k = -\frac{1}{6}$.

Example 3:

$$3kx + 5 = 9$$
$$3k(2) + 5 = 9$$
$$6k = 9 - 5$$
$$k = \frac{4}{6}$$

Answer / *Jawapan*: $k =$ ....$\frac{4}{6}$..

In the example above, the candidates have substituted correctly $x = 2$ and $\frac{dy}{dx} = 9$ into the equation $\frac{dy}{dx} = 3kx + 5$. However they did not simplify the final answer and left it as $\frac{4}{6}$.

Example 4:

$$\frac{dy}{dx} = 3kx + 5$$
$$m = 9$$
$$\therefore \quad 3k = 9$$
$$k = 3$$

In the example above, the candidates have mistook $\frac{dy}{dx} = 3kx + 5$ as the equation of a straight line in the gradient form $y = mx + c$. They further equated $3k$ (the gradient obtained from the candidates' equation) to 9 and obtained the wrong answer, $k = 3$.

QUESTION 20

Most candidates are able to differentiate $y = x^2 - 32x + 64$ correctly. Some are able to apply the concept of a minimum point. They are also some who used 'completing the square' method to find *p*. Only a few candidates used the formula $x = -\frac{b}{2a}$.

Example 1:

$$y = x^2 - 32x + 64$$
$$\frac{dy}{dx} = 2x - 32$$
$$\frac{d^2y}{dx^2} = 2 \ \text{(min.)}$$
$$\frac{dy}{dx} = 0, \quad 2x - 32 = 0$$
$$2x = 32$$
$$x = 16$$

Answer / *Jawapan*: $p = \cancel{2}\ 16$

In the example above, the candidates used $\frac{dy}{dx} = 0$ at a minimum point to find the value of p.

Example 2:

$$\cancel{y = x^2 + 32x + 64}$$

$$y = x^2 - 32x + 64$$

$$= x^2 - 32x + \left(-\frac{32}{2}\right)^2 - \left(-\frac{32}{2}\right)^2 + 64$$

$$= (x - 16)^2 - 192$$

$$x - 16 = 0$$

$$x = 16$$

$$x = p$$

$$p = 16$$

Answer / *Jawapan*: $p =$ 16.

In the example above, the candidates have used 'completing the square' method perfectly. Subsequently, they have equated $x - 16$ to 0, hence obtained $p = 16$.

Example 3:

$$x = -\left(\frac{-32}{2}\right)$$

$$x = 16$$

$$\therefore p = 16.$$

In this example, the candidates have used the formula $x = -\dfrac{b}{2a}$ to find p.

The following examples are the common mistakes made by the candidates.

Example 1:

$$x = p$$

$$\therefore y = p^2 - 32p + 64$$

$$p = \frac{-(-32) \pm \sqrt{(-32)^2 - 4(1)(64)}}{2(1)}$$

$$= \frac{32 \pm 27.71}{2(1)}$$

$$p = 29.86$$

In the example above, the candidates have applied the incorrect concept. They assumed that p is the root of the equation $y = x^2 - 32x + 64$. Hence, solved for p using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example 2:

$$y = x^2 - 32x + 64$$
$$\frac{dy}{dy} = 2x - 32$$

In the example above, the candidates stopped at $\dfrac{dy}{dx} = 2x - 32$. They are unable to apply the concept of the minimum point i.e. $\dfrac{dy}{dx} = 0$.

Example 3:

$$\begin{aligned} y &= x^2 - 32x + 64 \\ &= x^2 - 32x + \left(\tfrac{32}{2}\right)^2 - \left(\tfrac{32}{2}\right)^2 + 64 \\ &= (x + 16)^2 - 256 + 64 \\ &= (x + 16)^2 - 320 \\ x &= -16 \end{aligned}$$

Answer / *Jawapan*: $p = -16$

In the example above, the candidates made mistakes while 'completing the square'. They should have written $(x - 16)^2 - \ldots$ instead of $(x + 16)^2 - \ldots$

Example 4:

$$\begin{aligned} y &= x^2 - 32x + 64 \\ &= x^2 - 32x + 256 - 256 + 64 \\ &= (x - 16)^2 - 192 \end{aligned}$$

Answer / *Jawapan*: $p = -192$

In the example above, the candidates made the incorrect deduction even though they have completed the square perfectly. They have deduced $p = -192$ instead of $p = 16$.

QUESTION 21

Majority of the candidates can answer part (a) correctly. Those in the Higher Achiever Group are able to apply the concept $\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$. They are able to find the value of the definite integral i.e. $\int_2^7 5dx = [5x]_2^7$ = 5(7) – 5(2)

Example:

a) $\int_7^2 h(x)dx = -\int_2^7 h(x)\,dx$

$= -3$ #

b) $\int_2^7 [5 - h(x)]\,dx = \cancel{8 - \int_2^7 h}$

$= \int_2^7 5\,dx - \int_2^7 h(x)\,dx$

$= [5x]_2^7 - 3$

$= (35 - 10) - 3 = 22$ #

In the example above, the candidates could write $\int_2^7 [5 - h(x)]dx$ as $\int_2^7 5dx - \int_2^7 h(x)dx$. Many of the candidates are unable to solve part (b) because they could not apply the concept $\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$.

The following are the examples where the above concept is not applied.

Example 1:

(a) $\int_2^7 h(x)\,dx = 3$

$\int_7^2 h(x)\,dx = \underline{\underline{-3}}$ #

b) $\int_2^7 [5 - h(x)]\,dx$

$= \left[5x - h(x)dx\right]_2^7$

$= [5(7) - h(x)dx] - [5(2) - h(x)dx]$

$= 35 - h(x)dx - 10 + h(x)dx$

$= \underline{\underline{25}}$ #

Answer / *Jawapan*: (*a*) -3

(*b*) 25

Example 2:

a) $\int_{7}^{2} h(x)\,dx = -3$

b) $\int_{2}^{7} [5 - h(x)]\,dx$

$= \int_{2}^{7} 5x - 3x\,dx$

$= \cancel{\int_{2}^{7} 5x} \left[5x - 3x\right]_{2}^{7}$

$= [5(7) - 5(2)] - 3(7) - 3(2)]$

$= (35 - 10) - (21 - 6)$

$= 25 - 15$

$= 10$

Answer / *Jawapan*: (*a*) -3

(*b*) 10

In the example above, the candidates failed to write $\int_{2}^{7} 5 - h(x)dx$ as $\int_{2}^{7} 5dx - \int_{2}^{7} h(x)dx$. Hence they are unable to obtain the correct answer.

QUESTION 22

Most candidates can interpret the meaning of "sum of the numbers is 60" and "sum of the squares of the numbers is 800". Some of them are able to use the formulae for mean and standard deviation correctly.

Example:

$\Sigma fx = 60$

$\Sigma fx^2 = 800$

$n = 5$

$\therefore$ a) $\frac{60}{5}$

$\mu = 12$

b) $\sigma^2 = \frac{800}{5} - 12^2$

$\sigma^2 = 16$

$\therefore \sigma = 4$

Answer / *Jawapan*: (*a*) 12

(b) 4

In the example above, the candidates have interpreted "sum of the numbers is 60" as $\sum x = 60$ and "sum of the squares of the numbers is 800" as $\sum x^2 = 800$. They have used the formula $\bar{x} = \frac{\sum x}{N}$ to find for the mean and $\sigma = \sqrt{\frac{\sum x^2}{N} - (\bar{x})^2}$ to find the standard deviation correctly.

The following examples are the common mistakes made by the candidates.

Example 1:

a) $= \frac{800}{60}$

$= 13.3$

b) $S = \sqrt{\frac{\sum x^2}{N} - \bar{x}^2}$

$= \sqrt{\frac{800}{5} - 13.3}$

$= \sqrt{160 - 13.3}$

$= \sqrt{146.7}$

$= 12.11$

In the example above, the candidates mistook the “sum of the numbers 60” to be the total number of data and “sum of the squares of the numbers 800” to be the sum of numbers.

Example 2:

a) $\bar{x} = \dfrac{\sum x}{N}$

$= \dfrac{60}{5}$

$= 12$

b) $\sigma = \dfrac{\sum x^2}{N} - (\text{mean})^2$

$= \dfrac{800}{5} - 144$

$= 16$

In the example above, the candidates mistook the formula for the variance i.e.:

$\sigma^2 = \dfrac{\sum x^2}{N} - (x)^2$ to be the formula for standard deviation.

QUESTION 23

Only the candidates from the High Achiever Group can answer this question correctly. Candidates can use the combination concept ^nC_r to answer part (a) and the permutation concept ^nP_r to answer part (b).

Example:

(a) No. of ways the team can be formed

$= {}^4C_2 \times {}^5C_3$

$= 6 \times 10$

$= \underline{60}$ #

(b) No. of ways the team members can be arranged.

$= 3! \times 3!$

$= \underline{36}$ #

Answer / *Jawapan*: (*a*) 16

(*b*) 36

In the example above, the candidates can write the respective combinations correctly for the selection of boys and girls as ${}^4C_2$ and ${}^5C_3$ respectively. They further applied the multiplication principle $r \times s$ to obtain the correct answer.
For part (b), the candidates have used the number of ways to arrange the 3 female players as 3! and the 2 male players with the group of female player as 3!. They also used the multiplication principle $r \times s$ to obtain the correct answer.

The following examples are the common mistakes made by the candidates.

Example1:

(a)

$= {}^4C_2 + {}^5C_3$

$= 6 + 10$

$= 16$

(b)

$= 3! \times 2!$

$= 12$

In the example above for part (a), the candidates did not apply the multiplication principle $r \times s$ even though they have found the number of selection of the male and female payers correctly.
For part (b), the candidates failed to see that $3! \times 2!$ has to be multiplied by 3 in order to obtain the correct total number of arrangements.

Example2:

b) $\underline{3}\ \underline{2}\ \underline{1}\ \underline{2}\ \underline{1} = 12$

a) ${}^4P_2 \times {}^5P_3 = 720$

Answer / *Jawapan*: (*a*) 720

(*b*) 12

In the example above for part (a), the candidates have used the concept of permutation instead of combination.

For part (b), the candidates again failed to see that $3 \times 2 \times 1 \times 2 \times 1$ has to be multiplied by 3 in order to obtain the correct total number of arrangements.

QUESTION 24

Only the candidates from the High Achiever Group can answer the question correctly. Candidates know that the question requires them to use the formula to find probability of a discrete random variable which has a binomial distribution.
They are able to determine the value of *p*, *q*, *n* and *r* from the question. Some of them can substitute correctly the values into the formula and able to use the scientific calculator efficiently.

Example:

(a) $n = 10$ $\quad P = {}^nC_r\, p^r q^{n-r}$

$p = \frac{1}{3}$ $\quad = {}^{10}C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^8$

$q = \frac{2}{3}$

$r = 2$ $\quad = 0.19509$

(b) $n = ?$ $\quad {}^nC_r\, p^r q^{n-r} = \frac{1}{243}$

$p = \frac{1}{3}$ $\quad 1\left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^0 = \frac{1}{243}$

$q = \frac{2}{3}$

$r = n$ $\quad \left(\frac{1}{3}\right)^r = \frac{1}{243}$

$\left(\frac{1}{3}\right)^r = \left(\frac{1}{3}\right)^5$

$n = r = 5$

Answer / *Jawapan*: (*a*) 0.19509

(*b*) *n* = 5

From the example above for the part (a), the candidates substituted the values $p = \frac{1}{3}$, $q = \frac{2}{3}$, $n = 10$, and $r = 2$ into the correct formula ${}^nC_r p^r q^{n-r}$. For part (b), the candidates have interpreted and have equated ${}^nC_n p^n q^0$ with $\frac{1}{243}$ correctly. They subsequently solved it to obtain the correct answer for *n*.

The following examples are the common mistakes made by the candidates.

Example1:

a) $P(X=2) = {}^{10}C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^8$

$= 45 \times \left(\frac{1}{9}\right)\left(\frac{256}{6561}\right)$

$= \frac{1280}{6561}$

$= 0.1950$

b) $P(x=r) = {}^{n}C_n \left(1/243\right)^n \left(242/243\right)^0$

$\frac{1}{243} = 1\left(1/243\right)^n$

$\frac{1}{243} = \left(\frac{1}{243}\right)^n$

$n = 1$

Answer / *Jawapan*: (*a*) 0.1950

(*b*) $n =$ 1

In the example above for part (a), the candidates' final answer are rounded off inaccurately . The candidates wrote 0.1950 instead of 0.1951.

In part (b), the candidates mistook the value of *p* as $\frac{1}{243}$ instead of $p = \frac{1}{3}$.

Example2:

a. $X \sim B\left(10, \frac{1}{3}\right)$

$P(X=2) = 10C2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^8$

$= 0.1951$

b. $X \sim B\left(n, \frac{1}{3}\right)$

$P(X=n) = {}^{n}Cn \left(\frac{1}{3}\right)n \left(\frac{2}{3}\right)^0 = \frac{1}{243}$

$\frac{1}{3}n = \frac{1}{243}$

$n = \frac{1}{81}$

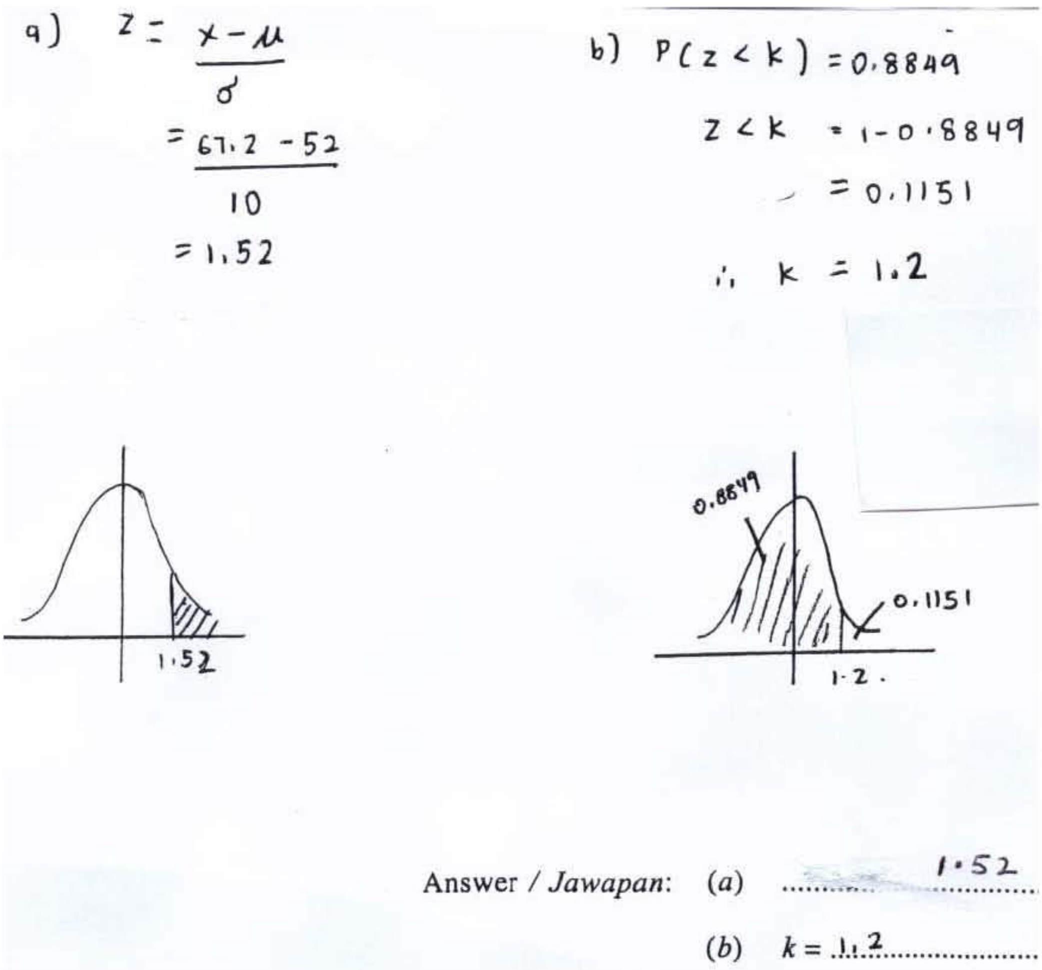
In the example above for part (a), the candidates wrote $\frac{1}{3}(n)$ instead of $\left(\frac{1}{3}\right)^n$. Hence, ended up with the incorrect answer $n = \frac{1}{81}$.

QUESTION 25

Many candidates try to answer this question but only the candidates from the High Achiever Group can answer this question correctly. Most candidates did not understand the requirement of section (b) of this question.

Most candidates who answered the question are able to determine the Z value, using the formula $Z = \frac{X-\mu}{\sigma}$. Competent candidates are able to identify the region in the standard normal distribution graph where its area represents the given probability.

Example:



a) $Z = \frac{x-\mu}{\sigma}$

$= \frac{67.2 - 52}{10}$

$= 1.52$

b) $P(z < k) = 0.8849$

$z < k = 1 - 0.8849$

$= 0.1151$

$\therefore k = 1.2$

Answer / *Jawapan*: (*a*) 1.52

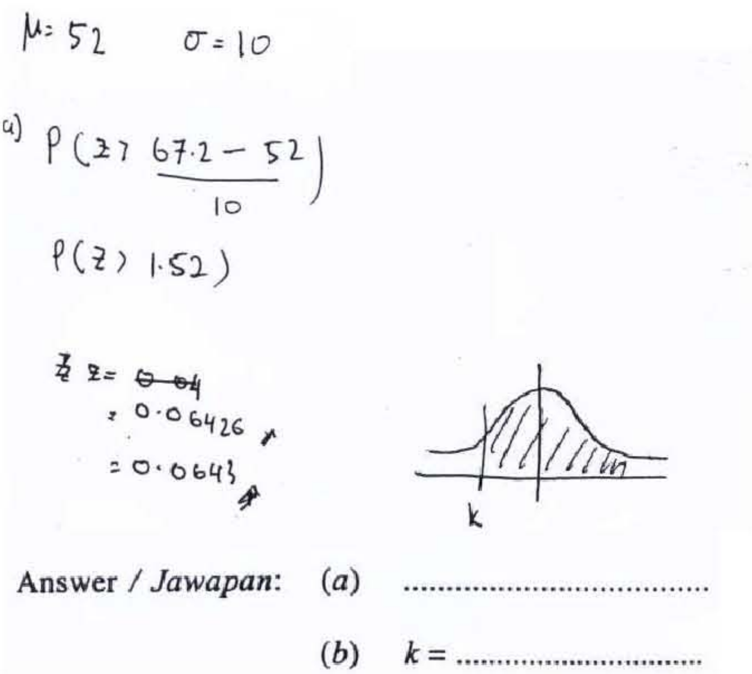
(*b*) $k =$ 1.2

In the example above, the candidates answered for part (a) by using the formula $Z = \frac{X-\mu}{\sigma}$, i.e. $Z = \frac{67.2-52}{10} = 1.2$.

In part (b), the candidates identified the correct region in the standard normal distribution graph that corresponds to $P(Z < k) = 0.1157$.

The following examples are the common mistakes made by the candidates.

Example 1:



$\mu = 52 \quad \sigma = 10$

a) $P\left(Z > \frac{67.2 - 52}{10}\right)$

$P(Z > 1.52)$

$Z = \cancel{0.04}$

$= 0.06426$

$= 0.0643$

k

Answer / *Jawapan*: (*a*) ....

(*b*) *k* = ...

In the example above for part (a), after obtaining the correct *Z*-score, the candidates failed to realize that they have obtained the correct answer. They went on further to find the probability:

$$P\left(Z > \frac{67.2 - 52}{10}\right) = P(Z > 1.52) = 0.0643$$

Example 2:

(a) $Z = \frac{X - \mu}{\sigma}$

$= \frac{67.2 - 52}{10}$

$= 1.52$

(b) $P(Z < k) = 0.8849$

$P(Z > k) = 0.1151$

$P\left(Z > \frac{k - 52}{10}\right) = 0.1151$

$\frac{k - 52}{10} = 1.20$

$k - 52 = 12$

$k = 64$

$P(Z < k) = 0.8849$

$P(Z > k) = 0.1151$

$\frac{k - \mu}{\sigma} = 1.2$

$\frac{k - 52}{10} = 1.2$

$k - 52 = 12$

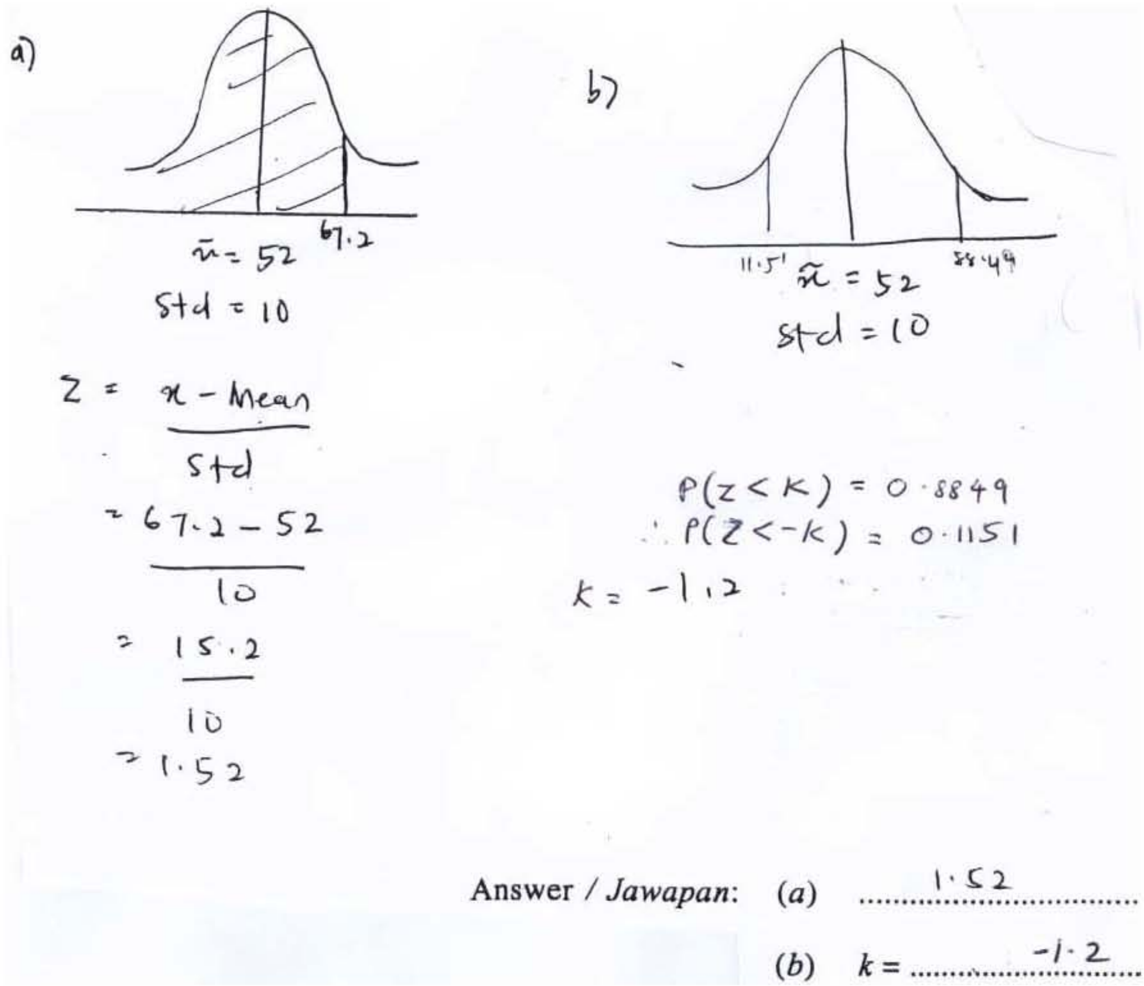
$k = 64$

Answer / *Jawapan*: (*a*) 1.52

(*b*) $k =$ 64

In the example above for part (b), the candidates mistook the answer they obtained 1.2 as the *X*-score. After obtaining the answer 1.2, they went further converting it into their *Z*-score.

Example 3:


a)

$\bar{x} = 52$ 67.2

std = 10

$Z = \dfrac{x - \text{Mean}}{\text{std}}$

$= \dfrac{67.2 - 52}{10}$

$= \dfrac{15.2}{10}$

$= 1.52$

b)

11.51 $\bar{x} = 52$ 88.49

std = 10

$P(z < k) = 0.8849$

$\therefore P(Z < -k) = 0.1151$

$k = -1.2$

Answer / *Jawapan*: (*a*) ......1.52......

(*b*) $k =$ ......-1.2......

In the example above, the candidates identified the incorrect region of the standard normal distribution for $P(Z > k) = 0.1151$. They mistook k to be on the left side of 0. Hence, obtaining $k = -1.2$.

SUGGESTIONS TO CANDIDATES

1. Improve the basic mathematical skills by doing a lot of exercises.
2. Attempt various types of questions in order to understand a topic well.
3. Understand the basic concepts taught in each topic since Paper 1 is testing the understanding and application of these basic concepts.
4. All the workings and solutions must be shown clearly in the working space provided.
5. Always remember to use decimal number up to four significant figures in the working.
6. Read the questions carefully and answer what is required in the question.
7. The scientific calculator must be used correctly and efficiently.
8. The answer in the working space must be transferred carefully and correctly to the answer space.
9. Use the correct symbols, formulae and terminologies.
10. Always check the answers obtained and make sure they are required in the question.

11. Underline or highlight the keywords in the questions such as ‘ value’, ‘values’ or a > 0 and give answers as required.
12. Do not miss out any important steps in calculation such as substitution in the correct formula, factorization, graphs, etc.
13. Make sure that the final answer is in the simplest form; if the answer involves decimal number, make sure the answer written is correct to four significant figures. If the question requires a certain level of accuracy, follow the instruction given.
14. Revise throughout the whole year and do the past year questions.
15. Pay attention in class and always ask the teachers whenever there is a problem.
16. Study in groups so that there is peer teaching and learning.

SUGGESTIONS TO TEACHERS:

1. Improve the students’ basic mathematical skills.
2. Guide the students to master the mathematical concepts and give examples on its application.
3. Always encourage students to attempt the basic questions to acquire total understanding of a topic.
4. Train students to show the working clearly in the working space.
5. Always remind students to use numbers approximated accurately to four significant figures in working.
6. Remind students to simplify the final answers or round off the final answer (in decimal form) to four significant figures.
7. Teach students to use the scientific calculator efficiently.
8. Teach students the techniques of answering questions.
9. Understand how the marks are awarded in order to guide students to answer accurately.
10. Problems must be diversified, starting from the simple to the highest level.
11. Teach according to the different levels of ability of the students. Teachers can apply the ‘Minimum Adequate Syllabus’ to the very weak students.
12. Identify the students’ weakness and do the necessary remedial teaching
13. Always motivate the students and give them a lot of encouragement.
14. Drill the students to answer questions within a specific time frame.